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4-Body Problem of Classical Electrodynamics - Derivation of Equations of Motion (I)

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Abstract

The primary purpose of the present paper is to continue the our previous investigations and apply the technique from the 2-body and 3-body problems of classical electrodynamics to the 4-body problem. First, we extend the problem for N charged particles based on our previous results and introducing radiation terms. Moreover, we justify the radiation time within the framework of non-standard analysis. The equations of motion are neutral type nonlinear differential equations with state dependent delays. In the next papers we prove the existence-uniqueness of a periodic solution. In this way we hope to explain the existence of Lithium atom and Hydrogen molecules.

Keywords: Classical electrodynamics, 4-body problem

1. Introduction

The primary goal of the present paper is to propose a general model of N-body problem with radiation terms. As a consequence, we derive equations describing the motion of 4 charged particles in the frame of classical electrodynamics. We proceed from the results [1, 2] modelling the interaction of N charged particles and add new form of the radiation terms introduced in [3, 4]. The present approach is already applied to 2- and 3-body problems [5, 6, 7, 8].

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In our model each particle is influenced by others (cf. [1]). From mathematical point of view the equation of motion of every particle (its right-hand side) contains the sum of the Lorentz force produced by all the other N-1 particles and the radiation term describing self-interaction. The base of our considerations is the Dirac reasoning presented in a relativistic invariant form [3, 4]. This leads to a nonlinear system of $4N$ equations of motion (cf. [2]):

$$m_k \frac{d\lambda_r^{(k)}}{ds_k} = \frac{e_k}{c^2}\left(F_{rl}^{(k,1)}\lambda_l^{(k)} + F_{rl}^{(k,2)}\lambda_l^{(k)} + \ldots + F_{rl}^{(k,k-1)}\lambda_l^{(k)} + F_{rl}^{(k,k+1)}\lambda_l^{(k)} + \ldots + F_{rl}^{(k,N)}\lambda_l^{(k)} + F_{rl}^{(k)rad}\lambda_l^{(k)}\right) \tag{1}$$

$(r = 1, 2, 3, 4; k = 1, 2, 3, \ldots, N)$ where the Einstein summation convention is valid for repeating l. The right-hand sides

$$F_{rl}^{(k,n)} = \frac{\partial \Phi_l^{(n)}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(n)}}{\partial x_l^{(k)}}$$

can be calculated by the retarded Lienard-Wiechert potentials

$$\Phi_r^{(n)} = -\frac{e_n \lambda_r^{(n)}}{\left\langle \lambda^{(n)}, \xi^{(kn)} \right\rangle_4}$$

following the results [13, 15, 18, 20]. The mathematical formulation of the retarded action is given in [13, 15, 18, 20]. The radiation term is defined as a half of the difference between both retarded and advanced potentials (cf. [10]):

$$F_{mn}^{(k)rad} = \frac{1}{2}\left[\left(\frac{\partial A_n^{(k)ret}}{\partial x_m^{(k)ret}} - \frac{\partial A_m^{(n)ret}}{\partial x_n^{(k)ret}}\right) - \left(\frac{\partial A_n^{(k)adv}}{\partial x_m^{(k)adv}} - \frac{\partial A_m^{(n)adv}}{\partial x_n^{(k)adv}}\right)\right]$$

where

$$A_n^{(k)ret} = -\frac{e_k \lambda_n^{(k)ret}}{\left\langle \lambda^{(k)ret}, \xi^{(k)ret} \right\rangle_4}$$

and

$$A_n^{(k)adv} = -\frac{e_k \lambda_n^{(k)adv}}{\left\langle \lambda^{(k)adv}, \xi^{(k)adv} \right\rangle_4}.$$

We have derived the radiation terms in a relativistic invariant form in [3, 4].

We proceed as in [5, 6, 7, 8] and obtain equations of motion for the 4-body problem. They are neutral type nonlinear differential equations with retarded arguments depending on the unknown trajectories (cf. [11, 12]). In the next paper we shall prove the existence of periodic orbits which confirms the Bohr-Sommerfeld hypotheses for stationary states [19]. It turns out that they are implied by classical electrodynamics unlike previous claims that they contradict it (cf. for instance [16]).

Section 1 of the paper is an Introduction. Here we extend the general formulation for N-body problem with radiation terms. In Section 2, a derivation of the explicit form of the equations of motion for 4-body problem is made. Every vector equation contains in the right-hand side the Lorentz force and a radiation term that shows the self-interaction of every moving particle. The Lorentz force is a sum of three terms each of which shows the influence of the other three particles on the first one. In Section 3, the formalism of the transition to the Euclidean coordinates is described. In Section 4, we reduce the system of equations of motion in a simplified form using suitable denotations. In Section 5, a transformation of the radiation terms on the base of non-standard analysis is made. Strictly speaking (in the frame of non-standard analysis) the electron size should be infinitely small because in the special relativity all objects are infinitely small (cf. [15]). This implies that the radiation time is infinitely small too (cf. [9, 14, 17]). Section 6 is a conclusion.

The equations of motion in the relativistic form for the 4-body problem are:

$$
\begin{aligned}
m_1\frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{c^2}\left(F_{rl}^{(12)}\lambda_l^{(1)}+F_{rl}^{(13)}\lambda_l^{(1)}+F_{rl}^{(14)}\lambda_l^{(1)}+F_{rl}^{(1)rad}\lambda_l^{(1)}\right),(r=1,2,3,4),\\
m_2\frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{c^2}\left(F_{rl}^{(21)}\lambda_l^{(2)}+F_{rl}^{(23)}\lambda_l^{(2)}+F_{rl}^{(24)}\lambda_l^{(2)}+F_{rl}^{(2)rad}\lambda_l^{(2)}\right),(r=1,2,3,4),\\
m_3\frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{c^2}\left(F_{rl}^{(31)}\lambda_l^{(3)}+F_{rl}^{(32)}\lambda_l^{(3)}+F_{rl}^{(34)}\lambda_l^{(3)}+F_{rl}^{(3)rad}\lambda_l^{(3)}\right),(r=1,2,3,4),\\
m_4\frac{d\lambda_r^{(4)}}{ds_4} &= \frac{e_4}{c^2}\left(F_{rl}^{(41)}\lambda_l^{(3)}+F_{rl}^{(42)}\lambda_l^{(3)}+F_{rl}^{(43)}\lambda_l^{(3)}+F_{rl}^{(4)rad}\lambda_l^{(4)}\right),(r=1,2,3,4).
\end{aligned}
\tag{2}
$$

Obviously, the number of equations in (2) is 16, while the unknown functions (trajectories) are 12. It is proved in [1, 2] that every 4-th equation is a consequence of the previous three ones. In this manner one obtains 12 equations of motion.

2. Derivation in Explicit Form of the Equations of Motion

Following denotations from [20] we mean by

$$\left(x_1^{(k)}(t),x_2^{(k)}(t),x_3^{(k)}(t),x_4^{(k)}=ict\right),(k=1,2,3,4)$$

the space-time coordinates of the charged particles. Latin indices run from 1 to 4 , while Greek - from 1 to 3 . The dot product in the Minkowski space is

$$\langle a,b\rangle_4=a_rb_r=\sum_{r=1}^{4}a_rb_r,$$

while in the 3-dimensional Eucleadian subspace

$$\langle a,b\rangle=a_\alpha b_\alpha=\sum_{\alpha=1}^{3}a_\alpha b_\alpha;$$

c is the vacuum speed of light; $m_k(k=1,2,3,4)$ are the proper masses of particles; $e_k(k=1,2,3,4)$ - their charges.

The elements of proper times are ds_k and

$$\gamma_k=1/\sqrt{1-\left(\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle/c^2\right)};\Delta_k=\sqrt{c^2-\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle};$$

$$\frac{d}{ds_k}=\frac{\gamma_k}{c}\frac{d}{dt};\lambda_\alpha^{(k)}=\frac{\gamma_k u_\alpha^{(k)}(t)}{c}=\frac{u_\alpha^{(k)}(t)}{\Delta_k}(\alpha=1,2,3),\lambda_4^{(k)}=i\gamma_k=\frac{ic}{\Delta_k}(k=1,2,3,4)$$

are the unit tangent vectors to the world lines

$$\lambda^{(k)}=\left(\lambda_1^{(k)},\lambda_2^{(k)},\lambda_3^{(k)},\lambda_4^{(k)}\right)=\left(\frac{\gamma_k u_1^{(k)}(t)}{c},\frac{\gamma_k u_2^{(k)}(t)}{c},\frac{\gamma_k u_2^{(k)}(t)}{c},i\gamma_k\right)=\left(\frac{u_1^{(1)}(t)}{\Delta_k},\frac{u_2^{(1)}(t)}{\Delta_k},\frac{u_3^{(1)}(t)}{\Delta_k},\frac{ic}{\Delta_k}\right)$$

$$\vec{u}^{(k)}=\left(u_1^{(k)}(t),u_2^{(k)}(t),u_3^{(k)}(t)\right)=\left(\dot{x}_1^{(k)}(t),\dot{x}_2^{(k)}(t),\dot{x}_3^{(k)}(t)\right).$$

The components of the accelerations are:

$$\frac{d\lambda^{(k)}}{ds_k}=\left(\frac{\gamma_k}{c}\frac{d}{dt}\frac{\gamma_k\lambda_1^{(k)}}{c},\frac{\gamma_k}{c}\frac{d}{dt}\frac{\gamma_k\lambda_2^{(k)}}{c},\frac{\gamma_k}{c}\frac{d}{dt}\frac{\gamma_k\lambda_3^{(k)}}{c},i\frac{\gamma_k}{c}\frac{d\gamma_k}{dt}\right)$$
$$=\left(\frac{1}{\Delta_k}\frac{d}{dt}\frac{u_1^{(k)}(t)}{\Delta_k},\frac{1}{\Delta_k}\frac{d}{dt}\frac{u_2^{(k)}(t)}{\Delta_k},\frac{1}{\Delta_k}\frac{d}{dt}\frac{u_3^{(k)}(t)}{\Delta_k},\frac{ic}{\Delta_k}\frac{d}{dt}\frac{1}{\Delta_k}\right).$$

The isotropic vectors $\xi^{(kn)}(k=1,2,3,4;n\neq k)$ are obtained as in [20] fixing any event on the world line of the k-th particle and drawing the light cone into the past. This cone intersects the world line of the n-th particle in any other (past) event. Since

$$\left\langle\xi^{(kn)},\xi^{(kn)}\right\rangle_4=0,$$

then the retarded functions $\tau_{kn}(t)$ should satisfy the following functional equations

$$\tau_{kn}(t)=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}=\frac{1}{c}\sqrt{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}(t)-x_\gamma^{(n)}\left(t-\tau_{kn}(t)\right]^2\right.}\quad(k=1,2,3,4;n\neq k)$$

and therefore

$$\xi^{(kn)}=\left(\xi_1^{(kn)},\xi_2^{(kn)},\xi_3^{(kn)},\xi_4^{(kn)}\right)$$
$$=\left(x_1^{(k)}(t)-x_1^{(n)}\left(t-\tau_{kn}\right),x_1^{(k)}(t)-x_1^{(n)}\left(t-\tau_{kn}\right),x_1^{(k)}(t)-x_1^{(n)}\left(t-\tau_{kn}\right),ic\tau_{kn}\right).$$

To obtain the radiation terms we proceed from the Dirac physical assumption [10], but we derive the radiation term in a relativistic invariant form [3, 4]. Consider the charge $e_k(k=1,2,3,4)$ describing the curve $L_k(k=1,2,3,4)$ in space-time. Let

$$A_k\left(x_1^{(k)}(t),x_2^{(k)}(t),x_3^{(k)}(t),ict\right)$$

be any event,

$$A_k^{ret}\left(x_1^{(k)}\left(\breve{t}_k\right),x_2^{(k)}\left(\breve{t}_k\right),x_3^{(k)}\left(\breve{t}_k\right),ic\breve{t}_k\right),\breve{t}_k<t$$

the intersection of $L_k(k=1,2,3,4)$ with the null-cone drawn into the past from $A_k(k=1,2,3,4)$ and

$$A_k^{adv}\left(x_1^{(k)}\left(\hat{t}_k\right),x_2^{(k)}\left(\hat{t}_k\right),x_3^{(k)}\left(\hat{t}_k\right),ic\hat{t}_k\right),\hat{t}_k>t$$

the intersection of $L_k(k=1,2,3,4)$ with the null-cone drawn into the future from $A_k(k=1,2,3,4)$. The components of the velocity vector to the world-line $L_k(k=1,2,3,4)$ at A_k^{ret} are

$$\lambda^{(k)ret}=\left(\lambda_1^{(k)ret},\lambda_2^{(k)ret},\lambda_3^{(k)ret},\lambda_4^{(k)ret}\right)=\left(\frac{u_1^{(k)}\left(\breve{t}_k\right)}{\Delta_k^{ret}},\frac{u_2^{(k)}\left(\breve{t}_k\right)}{\Delta_k^{ret}},\frac{u_3^{(k)}\left(\breve{t}_k\right)}{\Delta_k^{ret}},\frac{ic}{\Delta_k^{\mathrm{ret}}}\right)$$
$$=\left(\frac{\vec{u}^{(k)}\left(\breve{t}_k\right)}{\Delta_k^{ret}},\frac{ic}{\Delta_k^{\mathrm{ret}}}\right),\quad\Delta_k^{ret}=\sqrt{c^2-\left\langle\vec{u}^{(k)}\left(\breve{t}_k\right),\vec{u}^{(k)}\left(\breve{t}_k\right)\right\rangle}$$

and $A_k^{ret}A_k$ is the isotropic vector

$$\xi^{(k)ret}=\left(\xi_1^{(k)ret},\xi_2^{(k)ret},\xi_3^{(k)ret},\xi_4^{(k)ret}\right).$$

We set $\tau_k^{ret}(t) = t - \check{t}_k > 0 \Rightarrow \check{t}_k = t - \tau_k^{ret}(t)$ and then

$$\xi^{(k)ret} = \left(x_1^{(k)}(t) - x_1^{(k)}\left(\check{t}_k\right), x_2^{(k)}(t) - x_2^{(k)}\left(\check{t}_k\right), x_3^{(k)}(t) - x_3^{(k)}\left(\check{t}_k\right), ic\left(t - \check{t}_k\right)\right).$$

In a similar way we introduce the velocity vector to the world-line $L_k (k = 1, 2, 3)$ at A_k^{adv} :

$$\lambda^{(k)adv} = \left(\lambda_1^{(k)adv}, \lambda_2^{(k)adv}, \lambda_3^{(k)adv}, \lambda_4^{(k)adv}\right) = \left(\frac{u_1^{(k)}\left(\hat{t}_k\right)}{\Delta_k^{adv}}, \frac{u_2^{(k)}\left(\hat{t}_k\right)}{\Delta_k^{adv}}, \frac{u_3^{(k)}\left(\hat{t}_k\right)}{\Delta_k^{adv}}, \frac{ic}{\Delta_k^{adv}}\right)$$
$$= \left(\frac{\vec{u}^{(k)}\left(\hat{t}_k\right)}{\Delta_k^{adv}}, \frac{ic}{\Delta_k^{adv}}\right), \quad \Delta_k^{adv} = \sqrt{c^2 - \left\langle \vec{u}^{(k)}\left(\hat{t}_k\right), \vec{u}^{(k)}\left(\hat{t}_k\right)\right\rangle}$$

and $A_k^{adv} A_k$ is the isotropic vector.

$$\xi^{(k)adv} = \left(\xi_1^{(k)adv}, \xi_2^{(k)adv}, \xi_3^{(k)adv}, \xi_4^{(k)adv}\right).$$

Setting

$$\tau_k^{adv}(t) = \hat{t}_k - t > 0 \Rightarrow \hat{t}_k = t + \tau_k^{adv}(t)$$

one obtains

$$\xi^{(k)adv} = \left(x_1^{(k)}\left(\hat{t}_k\right) - x_1^{(k)}(t), x_2^{(k)}\left(\hat{t}_k\right) - x_2^{(k)}(t), x_3^{(k)}\left(\hat{t}_k\right) - x_3^{(k)}(t), ic\left(\hat{t}_k - t\right)\right).$$

Define the radiation terms as a half of the difference between both retarded and advanced potentials (cf. [10]):

$$F_{mn}^{(k)rad} = \frac{1}{2}\left[\left(\frac{\partial A_n^{(k)ret}}{\partial x_m^{(k)ret}} - \frac{\partial A_m^{(n)ret}}{\partial x_n^{(k)ret}}\right) - \left(\frac{\partial A_n^{(k)adv}}{\partial x_m^{(k)adv}} - \frac{\partial A_m^{(n)adv}}{\partial x_n^{(k)adv}}\right)\right]$$

where

$$A_n^{(k)ret} = -\frac{e_k \lambda_n^{(k)ret}}{\left\langle \lambda^{(k)ret}, \xi^{(k)ret}\right\rangle_4}$$

and

$$A_n^{(k)adv} = -\frac{e_k \lambda_n^{(k)adv}}{\left\langle \lambda^{(k)adv}, \xi^{(k)adv}\right\rangle_4}.$$

Then the equations of motion become $(r = 1, 2, 3, 4)$

$$m_1 \frac{d\lambda_r^{(1)}}{ds_1} = \frac{e_1}{c^2}\left(F_{rl}^{(12)}\lambda_l^{(1)} + F_{rl}^{(13)}\lambda_l^{(1)} + F_{rl}^{(14)}\lambda_l^{(1)} + \frac{1}{2}\left[\frac{\partial A_l^{(1)ret}}{\partial x_r^{(1)ret}} - \frac{\partial A_r^{(1)ret}}{\partial x_l^{(1)ret}} - \left(\frac{\partial A_l^{(1)adv}}{\partial x_r^{(1)adv}} - \frac{\partial A_r^{(1)adv}}{\partial x_l^{(1)adv}}\right)\right]\lambda_l^{(1)}\right),$$
$$m_2 \frac{d\lambda_r^{(2)}}{ds_2} = \frac{e_2}{c^2}\left(F_{rl}^{(21)}\lambda_l^{(2)} + F_{rl}^{(23)}\lambda_l^{(2)} + F_{rl}^{(24)}\lambda_l^{(2)} + \frac{1}{2}\left[\frac{\partial A_l^{(2)ret}}{\partial x_r^{(2)ret}} - \frac{\partial A_r^{(2)ret}}{\partial x_l^{(2)ret}} - \left(\frac{\partial A_l^{(2)adv}}{\partial x_r^{(2)adv}} - \frac{\partial A_r^{(2)adv}}{\partial x_l^{(2)adv}}\right)\right]\lambda_l^{(2)}\right),$$
$$m_3 \frac{d\lambda_r^{(3)}}{ds_3} = \frac{e_3}{c^2}\left(F_{rl}^{(31)}\lambda_l^{(3)} + F_{rl}^{(32)}\lambda_l^{(3)} + F_{rl}^{(32)}\lambda_l^{(3)} + \frac{1}{2}\left[\frac{\partial A_l^{(3)ret}}{\partial x_r^{(3)ret}} - \frac{\partial A_r^{(3)ret}}{\partial x_l^{(3)ret}} - \left(\frac{\partial A_l^{(3)adv}}{\partial x_r^{(3)adv}} - \frac{\partial A_r^{(3)adv}}{\partial x_l^{(3)adv}}\right)\right]\lambda_l^{(3)}\right),$$
$$m_4 \frac{d\lambda_r^{(4)}}{ds_4} = \frac{e_4}{c^2}\left(F_{rl}^{(41)}\lambda_l^{(4)} + F_{rl}^{(42)}\lambda_l^{(4)} + F_{rl}^{(43)}\lambda_l^{(4)} + \frac{1}{2}\left[\frac{\partial A_l^{(4)ret}}{\partial x_r^{(4)ret}} - \frac{\partial A_r^{(4)ret}}{\partial x_l^{(4)ret}} - \left(\frac{\partial A_l^{(4)adv}}{\partial x_r^{(4)adv}} - \frac{\partial A_r^{(4)adv}}{\partial x_l^{(4)adv}}\right)\right]\lambda_l^{(4)}\right).$$

Generalizing the formalism from [5, 6, 7, 8] we reach the following system of 16 equations ($k = 1,2,3,4; \alpha = 1,2,3,4$) :

$$\begin{aligned}\frac{d\lambda_\alpha^{(k)}}{ds_k} &= \sum_{n=1,n\neq k}^{4} \frac{e_k e_n}{m_k c^2}\left\{\frac{\xi_\alpha^{(kn)}\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4 - \lambda_\alpha^{(n)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(n)}}{ds_n},\xi^{(kn)}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^2}\left[\frac{d\lambda_\alpha^{(n)}}{ds_n}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4 - \xi_\alpha^{(kn)}\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right]\right\}\\
&+\frac{e_k^2}{2m_k c^2}\left[\frac{\xi_\alpha^{(k)ret}\left\langle\lambda^{(k)},\lambda^{(k)ret}\right\rangle_4 - \lambda_\alpha^{(k)ret}\left\langle\xi^{(k)ret},\lambda^{(k)}\right\rangle_4}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(k)ret}}{ds_{ret}},\xi^{(k)ret}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^2}\left(\frac{d\lambda_\alpha^{(k)ret}}{ds_{ret}}\left\langle\xi^{(k)ret},\lambda^{(k)}\right\rangle_4 - \xi_\alpha^{(k)ret}\left\langle\lambda^{(k)},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\right)\right]\\
&-\frac{e_k^2}{2m_k c^2}\left[\frac{\xi_\alpha^{(k)adv}\left\langle\lambda^{(k)},\lambda^{(k)adv}\right\rangle_4 - \lambda_\alpha^{(k)adv}\left\langle\xi^{(k)adv},\lambda^{(k)}\right\rangle_4}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(k)adv}}{ds_{adv}},\xi^{(k)adv}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^2}\left(\frac{d\lambda_\alpha^{(k)adv}}{ds_{adv}}\left\langle\xi^{(k)adv},\lambda^{(k)}\right\rangle_4 - \xi_\alpha^{(k)adv}\left\langle\lambda^{(k)},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\right)\right], \qquad (3)\end{aligned}$$

$$\begin{aligned}\frac{d\lambda_4^{(k)}}{ds_k} &= \sum_{n=1,n\neq k}^{4} \frac{e_k e_n}{m_k c^2}\left\{\frac{\xi_4^{(kn)}\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4 - \lambda_4^{(n)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left(1+\left\langle\frac{d\lambda^{(n)}}{ds_n},\xi^{(kn)}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^2}\left[\frac{d\lambda_4^{(n)}}{ds_n}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4 - \xi_4^{(kn)}\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right]\right\}\\
&+\frac{e_k^2}{2m_k c^2}\left[\frac{\xi_4^{(k)ret}\left\langle\lambda^{(k)},\lambda^{(k)ret}\right\rangle_4 - \lambda_4^{(k)ret}\left\langle\xi^{(k)ret},\lambda^{(k)}\right\rangle_4}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^3}\left(1+\left\langle\xi^{(k)ret},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^2}\left(\left\langle\xi^{(k)ret},\lambda^{(k)}\right\rangle_4\frac{d\lambda_4^{(k)ret}}{ds_{ret}} - \left\langle\lambda^{(k)},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4\xi_4^{(k)ret}\right)\right]\\
&-\frac{e_k^2}{2m_k c^2}\left[\frac{\xi_\alpha^{(k)adv}\left\langle\lambda^{(k)},\lambda^{(k)adv}\right\rangle_4 - \lambda_\alpha^{(k)adv}\left\langle\xi^{(k)adv},\lambda^{(k)}\right\rangle_4}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^3}\left(1+\left\langle\xi^{(k)adv},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\right)\right.\\
&\left.+\frac{1}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^2}\left(\left\langle\xi^{(k)adv},\lambda^{(k)}\right\rangle_4\frac{d\lambda_4^{(k)adv}}{ds_{adv}} - \left\langle\lambda^{(k)},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4\xi_4^{(k)adv}\right)\right].\end{aligned}$$

3. Transition to Euclidean Coordinates

Since $\xi^{(kn)}, \xi^{(k)ret}, \xi^{(k)adv}$ are isotropic 4-vectors we obtain

$$\left\langle\xi^{(kn)},\xi^{(kn)}\right\rangle_4 = 0, \left\langle\xi^{(k)ret},\xi^{(k)ret}\right\rangle_4 = 0, \left\langle\xi^{(k)adv},\xi^{(k)adv}\right\rangle_4 = 0$$

Therefore, considering their coordinates

$$\xi^{(kn)}=\left(x_1^{(k)}(t)-x_1^{(n)}\left(t-\tau_{kn}\right),x_2^{(k)}(t)-x_2^{(n)}\left(t-\tau_{kn}\right),x_3^{(k)}(t)-x_3^{(n)}\left(t-\tau_{kn}\right),ic\tau_{kn}\right);$$

$$\xi^{(k)ret}=\left(x_1^{(k)}(t)-x_1^{(k)}\left(t-\tau_k^{ret}\right),x_2^{(k)}(t)-x_2^{(k)}\left(t-\tau_k^{ret}\right),x_3^{(k)}(t)-x_3^{(k)}\left(t-\tau_k^{ret}\right),ic\tau_k^{ret}\right);$$

$$\xi^{(k)adv}=\left(x_1^{(k)}\left(t+\tau_k^{adv}\right)-x_1^{(k)}(t),x_2^{(k)}\left(t+\tau_k^{adv}\right)-x_2^{(k)}(t),x_3^{(k)}\left(t+\tau_k^{adv}\right)-x_3^{(k)}(t),ic\tau_k^{adv}\right)$$

we obtain $\tau_{kn}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle},\ \tau_k^{ret}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(k)ret},\vec{\xi}^{(k)ret}\right\rangle},\ \tau_k^{adv}=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle}.$

In this way the solutions the last equations define the retarded and advanced functions $\tau_{kn},\tau_k^{\mathrm{ret}},\tau_k^{adv}$. Differentiating the functions $t=t\left(t_{kn}\right)$ we obtain

$$\frac{dt}{dt_{kn}}=\frac{c\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle}=\frac{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle}=D_{kn}.$$

Then for the velocity vectors we have

$$\lambda^{(n)}=\left(\frac{u_1^{(n)}\left(t-\tau_{kn}\right)}{\Delta_{kn}},\frac{u_2^{(n)}\left(t-\tau_{kn}\right)}{\Delta_{kn}},\frac{u_3^{(n)}\left(t-\tau_{kn}\right)}{\Delta_{kn}},\frac{ic}{\Delta_{kn}}\right)=\left(\frac{\vec{u}^{(n)}\left(t-\tau_{kn}\right)}{\Delta_{kn}},\frac{ic}{\Delta_{kn}}\right)$$

where

$$\Delta_{kn}=\sqrt{c^2-\left\langle\vec{u}^{(n)}\left(t-\tau_{kn}\right),\vec{u}^{(n)}\left(t-\tau_{kn}\right)\right\rangle}.$$

For the acceleration vectors and scalar products in the Minkowski space we have

$$\frac{d\lambda_\alpha^{(k)}}{ds_k}=\frac{1}{\Delta_k}\frac{d}{dt}\left(\frac{u_\alpha^{(k)}(t)}{\Delta_k}\right)=\frac{\dot{u}_\alpha^{(k)}(t)}{\Delta_k^2}+\frac{u_\alpha^{(k)}(t)\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4}$$

$$;\frac{d\lambda_4^{(k)}}{ds_k}=\frac{ic}{\Delta_k}\frac{d}{dt}\left(\frac{1}{\Delta_k}\right)=\frac{ic\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4};$$

$$\frac{d\lambda_\alpha^{(n)}}{ds_n}=D_{kn}\left(\frac{\dot{u}_\alpha^{(n)}\left(t-\tau_{kn}\right)}{\Delta_{kn}^2}+\frac{u_\alpha^{(n)}\left(t-\tau_{kn}\right)\left\langle\vec{u}^{(n)}\left(t-\tau_{kn}\right),\dot{\vec{u}}^{(n)}\left(t-\tau_{kn}\right)\right\rangle}{\Delta_{kn}^4}\right)$$

$$;\frac{d\lambda_4^{(n)}}{ds_n}=\frac{icD_{kn}\left\langle\vec{u}^{(n)}\left(t-\tau_{kn}\right),\dot{\vec{u}}^{(n)}\left(t-\tau_{kn}\right)\right\rangle}{\Delta_{kn}^4}$$

$$\left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(n)}\left(t-\tau_{kn}\right)\right\rangle-c^2}{\Delta_k\Delta_{kn}};\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k}$$

$$\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4=\frac{\left\langle\vec{u}^{(n)}\left(t-\tau_{kn}\right),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_{kn}};$$

$$\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4=D_{kn}\left(\frac{\left\langle\vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}\left(t-\tau_{kn}\right)\right\rangle}{\Delta_{kn}^2}+\frac{\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\left(t-\tau_{kn}\right)\right\rangle-c^2\tau_{kn}}{\Delta_{kn}^4}\left\langle\vec{u}^{(n)}\left(t-\tau_{kn}\right),\dot{\vec{u}}^{(n)}\left(t-\tau_{kn}\right)\right\rangle\right);$$

$$\left\langle \lambda^{(k)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 = \frac{D_{kn}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}(t), \dot{\vec{u}}^{(n)}(t-\tau_{kn}) \right\rangle}{\Delta_{kn}^2} \right.$$
$$\left. + \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(n)}(t-\tau_{kn}) \right\rangle - c^2\tau_{kn}}{\Delta_{kn}^4} \left\langle \vec{u}^{(n)}(t-\tau_{kn}), \dot{\vec{u}}^{(n)}(t-\boldsymbol{\tau}_{kn}) \right\rangle \right)$$

For the retarded part of radiation term considering $t = t\left(\check{t}_k\right)$ we differentiate

$$t - \check{t}_k = \frac{1}{c}\sqrt{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}(t) - x_\gamma^{(k)}\left(\check{t}_k\right)\right]^2}$$

with respect to $\check{t}_k$ and obtain

$$D_k^{ret} \equiv \frac{dt}{d\check{t}_k} = \frac{c\sqrt{\left\langle \vec{\xi}^{(k)ret}, \vec{\xi}^{(k)ret} \right\rangle} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle}{c\sqrt{\left\langle \vec{\xi}^{(k)ret}, \vec{\xi}^{(k)ret} \right\rangle} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)} \right\rangle} = \frac{c^2\tau_k^{\mathrm{ret}} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}\left(t - \boldsymbol{\tau}_k^{ret}\right) \right\rangle}{c^2\tau_k^{\mathrm{ret}} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}(t) \right\rangle}.$$

In a similar way from $t = t\left(\check{t}_k\right)$ or

$$\hat{t}_k - t = \frac{1}{c}\sqrt{\sum_{\gamma=1}^{3}\left[x_\gamma^{(k)}\left(\hat{t}_k\right) - x_\gamma^{(k)}(t)\right]^2}$$

differentiate with respect to $\hat{t}_k$ we obtain

$$D_k^{adv} \equiv \frac{dt}{d\hat{t}_k} = \frac{c\sqrt{\left\langle \vec{\xi}^{(k)adv}, \vec{\xi}^{(k)adv} \right\rangle} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle}{c\sqrt{\left\langle \vec{\xi}^{(k)adv}, \vec{\xi}^{(k)adv} \right\rangle} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)} \right\rangle} = \frac{c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}\left(t + \tau_k^{adv}\right) \right\rangle}{c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}(t) \right\rangle}.$$

Further on we have

$$\left\langle \lambda^{(k)}, \lambda^{(k)ret} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}\left(t - \tau_k^{ret}\right) \right\rangle - c^2}{\Delta_k \Delta_k^{\mathrm{ret}}}; \left\langle \lambda^{(k)}, \lambda^{(k)adv} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}\left(t + \tau_k^{adv}\right) \right\rangle - c^2}{\Delta_k \Delta_k^{adv}};$$
$$\left\langle \xi^{(k)ret}, \lambda^{(k)} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{\xi}^{(k)ret} \right\rangle - c^2\tau_k^{ret}}{\Delta_k}; \left\langle \xi^{(k)adv}, \lambda^{(k)} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t), \vec{\xi}^{(k)adv} \right\rangle - c^2\tau_k^{adv}}{\Delta_k};$$
$$\left\langle \xi^{(k)ret}, \lambda^{(k)ret} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)ret}\left(t - \tau_k^{ret}\right), \vec{\xi}^{(k)ret} \right\rangle - c^2\tau_k^{ret}}{\Delta_k^{\mathrm{ret}}}; \left\langle \xi^{(k)adv}, \lambda^{(k)adv} \right\rangle_4$$
$$= \frac{\left\langle \vec{u}^{(k)adv}\left(t + \tau_k^{adv}\right), \vec{\xi}^{(k)adv} \right\rangle - c^2\tau_k^{adv}}{\Delta_k^{adv}};$$

$$\frac{d}{ds_{ret}}=\frac{1}{\Delta_k^{\mathrm{ret}}}\frac{d}{d\breve{t}}=\frac{1}{\Delta_k^{ret}}\frac{dt}{d\breve{t}_k}\frac{d}{dt}=\frac{1}{\Delta_k^{\mathrm{ret}}}D_k^{ret}\frac{d}{dt};\ \frac{d}{ds_{adv}}=\frac{1}{\Delta_k^{adv}}\frac{d}{d\breve{t}}=\frac{1}{\Delta_k^{adv}}\frac{dt}{d\breve{t}_k}\frac{d}{dt}=\frac{1}{\Delta_k^{adv}}D_k^{adv}\frac{d}{dt};$$

$$\frac{d\lambda_\alpha^{(k)ret}}{ds_{\mathrm{ret}}}=D_k^{ret}\left(\frac{\dot{u}_\alpha^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)}{\left(\Delta_k^{\mathrm{ret}}\right)^2}+\frac{u_\alpha^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\left\langle\vec{u}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right),\dot{\vec{u}}^{(k)}\left(t-\tau_k^{ret}\right)\right\rangle}{\left(\Delta_k^{\mathrm{ret}}\right)^4}\right);$$

$$\frac{d\lambda_4^{(k)ret}}{ds_{\mathrm{ret}}}=\frac{icD_k^{ret}\left\langle\vec{u}^{(k)}\left(t-\tau_k^{ret}\right),\dot{\vec{u}}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle}{\left(\Delta_k^{\mathrm{ret}}\right)^4};$$

$$\frac{d\lambda_\alpha^{(k)adv}}{ds_{adv}}=D_k^{adv}\left(\frac{\dot{u}_\alpha^{(k)}\left(t+\tau_k^{adv}\right)}{\left(\Delta_k^{adv}\right)^2}+\frac{u_\alpha^{(k)}\left(t+\tau_k^{adv}\right)\left\langle\vec{u}^{(k)}\left(t+\tau_k^{adv}\right),\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle}{\left(\Delta_k^{adv}\right)^4}\right);$$

$$\frac{d\lambda_4^{(k)adv}}{ds_{adv}}=\frac{icD_k^{adv}\left\langle\vec{u}^{(k)}\left(t+\tau_k^{adv}\right),\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle}{\left(\Delta_k^{adv}\right)^4};$$

$$\left\langle\xi^{(k)ret},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4=D_k^{\mathrm{ret}}\left(\frac{\left\langle\vec{\xi}^{(k)ret},\dot{\vec{u}}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle}{\left(\Delta_k^{\mathrm{ret}}\right)^2}\right.$$
$$\left.+\frac{\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle-c^2\tau_k^{\mathrm{ret}}}{\left(\Delta_k^{\mathrm{ret}}\right)^4}\left\langle\vec{u}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right),\dot{\vec{u}}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle\right);$$

$$\left\langle\lambda^{(k)},\frac{d\lambda^{(k)ret}}{ds_{ret}}\right\rangle_4=\frac{D_k^{ret}}{\Delta_k}\left(\frac{\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle}{\left(\Delta_k^{\mathrm{ret}}\right)^2}\right.$$
$$\left.+\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle-c^2\tau_k^{\mathrm{ret}}}{\left(\Delta_k^{\mathrm{ret}}\right)^4}\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}\left(t-\tau_k^{\mathrm{ret}}\right)\right\rangle\right);$$

$$\left\langle\xi^{(k)adv},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4=D_k^{adv}\left(\frac{\left\langle\vec{\xi}^{(k)adv},\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle}{\left(\Delta_k^{adv}\right)^2}\right.$$
$$\left.+\frac{\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle-c^2\tau_k^{adv}}{\left(\Delta_k^{adv}\right)^4}\left\langle\vec{u}^{(k)}\left(t+\tau_k^{adv}\right),\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle\right);$$

$$\left\langle\lambda^{(k)},\frac{d\lambda^{(k)adv}}{ds_{adv}}\right\rangle_4=\frac{D_k^{adv}}{\Delta_k}\left(\frac{\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle}{\left(\Delta_k^{adv}\right)^2}\right.$$
$$\left.+\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle-c^2\tau^{(k)adv}}{\left(\Delta_k^{ad\nu}\right)^4}\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle\right);$$

$$H_{kn}=1+D_{kn}\left(\frac{\left\langle\vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^{2}}+\frac{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^{2}\tau_{kn}\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^{4}}\right),(k=1,2,3,4),n\neq k;$$

$$H_{k}^{\mathrm{ret}}=1+D_{k}^{\mathrm{ret}}\left(\frac{\left\langle\vec{\xi}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_{k}^{\mathrm{ret}}\right)^{2}}+\frac{\left(\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle-c^{2}\tau_{k}^{\mathrm{ret}}\right)\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_{k}^{\mathrm{ret}}\right)^{4}}\right)$$

$$H_{k}^{adv}=1+D_{k}^{adv}\left(\frac{\left\langle\vec{\xi}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_{k}^{adv}\right)^{2}}+\frac{\left(\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle-c^{2}\tau_{k}^{adv}\right)\left\langle\vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_{k}^{adv}\right)^{4}}\right).$$

Then the equations of motion (3) become $(k=1,2,3,4;\alpha=1,2,3)$:

$$\frac{1}{\Delta_{k}^{2}}\dot{u}_{\alpha}^{(k)}+\frac{u_{\alpha}^{(k)}}{\Delta_{k}^{4}}\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)}\right\rangle=\sum_{n=1,n\neq k}^{4}\frac{e_{n}e_{k}}{m_{k}c^{2}}\left\{\frac{\frac{\left(c^{2}-\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)}{\Delta_{k}\Delta_{kn}}\xi_{\alpha}^{(kn)}-\frac{u_{\alpha}^{(n)}}{\Delta_{kn}}\frac{\left(c^{2}\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle\right)}{\Delta_{k}}}{\left(\frac{\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^{2}\tau_{kn}}{\Delta_{kn}}\right)^{3}}H_{kn}\right.\tag{4}$$

$$+\Delta_{kn}^{2}\left[\frac{\frac{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-c^{2}\tau_{kn}\right)}{\Delta_{k}}D_{kn}\left(\frac{\dot{u}_{\alpha}^{(n)}}{\Delta_{kn}^{2}}+\frac{u_{\alpha}^{(n)}\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^{4}}\right)}{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^{2}\tau_{kn}\right)^{2}}-\frac{\xi_{\alpha}^{(kn)}\frac{D_{kn}}{\Delta_{k}}\left(\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^{2}}+\frac{\left(\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle-c^{2}\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^{4}}\right)}{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^{2}\tau_{kn}\right)^{2}}\right]\Bigg\}$$

$$+\frac{e_{k}^{2}}{2m_{k}c^{2}}\left\{\frac{\xi_{\alpha}^{(k)ret}\frac{\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle-c^{2}}{\Delta_{k}\Delta_{k}^{\mathrm{ret}}}-u_{\alpha}^{(k)ret}\frac{\vec{u}^{(k)},\vec{\xi}^{(k)ret}\rangle-c^{2}\tau_{k}^{\mathrm{let}}}{\Delta_{k}\Delta_{k}^{\mathrm{ret}}}}{\left(\frac{\left\langle\vec{u}^{(k)ret},\vec{\xi}^{(k)ret}\right\rangle-c^{2}\tau_{k}^{ret}}{\Delta_{k}^{\mathrm{vet}}}\right)^{3}}H_{k}^{\mathrm{ret}}\right.$$

$$+\Delta_{(k)ret}^{2}\left[\frac{\frac{\left(\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle-c^{2}\tau_{k}^{ret}\right)D_{k}^{\mathrm{ret}}}{\Delta_{k}}\left(\frac{\dot{u}_{\alpha}^{(p)ret}}{\left(\Delta_{k}^{ret}\right)^{2}}+\frac{u_{\alpha}^{(p)r}\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_{k}^{ret}\right)^{4}}\right)}{\left(c^{2}\tau_{k}^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^{2}}\right.$$

$$\left.\left.-\frac{\xi_{\alpha}^{(k)ret}\frac{D_{k}^{ret}}{\Delta_{k}}\left(\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_{k}^{ret}\right)^{2}}+\frac{\left(\left\langle\overrightarrow{\mathrm{u}}^{(k)},\overrightarrow{\mathrm{u}}^{(k)ret}\right\rangle-c^{2}\right)\left\langle\overrightarrow{\mathrm{u}}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_{k}^{ret}\right)^{4}}\right)}{\left(c^{2}\tau_{k}^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^{2}}\right]\right\}$$

$$-\frac{e_{k}^{2}}{2m_{k}c^{2}}\left\{\frac{\xi_{\alpha}^{(k)adv}\frac{\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle-c^{2}}{\Delta_{k}\Delta_{k}^{adv}}-\frac{u_{\alpha}^{(k)adv}}{\Delta_{k}^{adv}}\frac{\left\langle\vec{u}^{(k)},\vec{\xi}^{(k)adv}\right\rangle-c^{2}\tau_{k}^{adv}}{\Delta_{k}}}{\left(\frac{\left\langle\vec{u}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle-c^{2}\tau_{k}^{adv}}{\Delta_{k}^{adv}}\right)^{3}}H_{k}^{adv}+\right.$$

$$+\Delta_{(k)adv}^{2}\left[\frac{\frac{\left\langle\vec{\xi}^{(p)adv},\vec{u}^{(k)}\right\rangle-c^{2}\tau_{k}^{adv}}{\Delta_{k}}D_{k}^{adv}\left(\frac{\dot{u}_{\alpha}^{(k)adv}}{\left(\Delta_{k}^{adv}\right)^{2}}+\frac{u_{\alpha}^{(k)adv}\left\langle\vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_{k}^{adv}\right)^{4}}\right)}{\left(c^{2}\tau_{k}^{adv}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^{2}}\right.$$

$$\left.\left.-\frac{\xi_{\alpha}^{(k)adv}\frac{D_{k}^{adv}}{\Delta_{k}}\left(\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_{k}^{adv}\right)^{2}}+\frac{\left(\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle-c^{2}\right)\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_{k}^{adv}\right)^{4}}\right)}{\left(c^{2}\tau_{k}^{adv}-\left\langle\vec{\xi}(k)adv,\vec{u}^{(k)adv}\right\rangle\right)^{2}}\right]\right\}\quad(\alpha=1,2,3);$$

$$\frac{ic}{\Delta_k^4}\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle = \sum_{n=1, n\neq k}^{4} \frac{e_k e_n}{m_k c^2} \left\{ \frac{\frac{\left(c^2 - \left\langle \vec{u}^{(k)}, \vec{u}^{(n)} \right\rangle\right)}{\Delta_k \Delta_{kn}} ic\tau_{kn} - \frac{ic}{\Delta_{kn}} \frac{\left(c^2\tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(k)} \right\rangle\right)}{\Delta_k}}{\left(\frac{c^2\tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle}{\Delta_{kn}}\right)^3} H_{kn} \right. \tag{5}$$

$$+ \Delta_{kn}^2 \left[\frac{\frac{\left\langle \xi^{(kn)}, \vec{u}^{(k)} \right\rangle - c^2\tau_{kn}}{\Delta_p} \frac{icD_{kn}}{\Delta_{pq}^4} \left\langle \vec{u}^{(n)}, \rightrightarrows \dot{\vec{u}}^{(n)} \right\rangle}{\left(\left\langle \vec{\xi}(kn), \vec{u}^{(n)} \right\rangle - c^2\tau_{kn}\right)^2} \right.$$

$$\left. \left. - \frac{ic\tau_{kn} \frac{D_{kn}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{\tau_{kn}}^2} + \frac{\left(\left\langle \vec{u}^{(k)}, \vec{u}^{(n)} \right\rangle - c^2\right)\left\langle \vec{u}^{(n)}, \overrightarrow{\dot{u}^{(n)}} \right\rangle}{\Delta_{kn}^4} \right)}{\left(\left\langle \vec{\xi}(kn), \vec{u}^{(n)} \right\rangle - c^2\tau_{kn}\right)^2} \right] \right\}$$

$$+ \frac{e_k^2}{2m_k c^2} \left\{ \frac{ic\tau_k^{\text{ret}} \frac{\left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret} \right\rangle - c^2}{\Delta_k \Delta_k^{\text{ret}}} - \frac{ic}{\Delta_k^{\text{ret}}} \frac{\left\langle \vec{u}^{(k)}, \vec{\xi}^{(k)ret} \right\rangle - c^2\tau_k^{\text{let}}}{\Delta_k}}{\left(\frac{\left\langle \vec{u}^{(k)ret}, \vec{\xi}^{(k)ret} \right\rangle - c^2\tau_k^{\text{ret}}}{\Delta_k^{\text{ret}}}\right)^3} H_k^{ret} \right.$$

$$+ \Delta_{(k)\ \text{ret}}^2 \left[\frac{\left[\frac{\left\langle \xi^{(k)ret}, \vec{u}^{(k)} \right\rangle - c^2\tau_k^{\text{ret}}}{\Delta_k} \frac{icD_k^{\text{ret}} \left\langle \vec{u}^{(k)ret}, \dot{\vec{u}}^{(k)ret} \right\rangle}{\left(\Delta_k^{\text{ret}}\right)^4} \right.}{\left(c^2\tau_k^{\text{ret}} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle\right)^2} \right.$$

$$\left. \left. - ic\tau_k^{\text{ret}} \frac{D_k^{\text{ret}}}{\Delta_k} \frac{\left(\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)ret} \right\rangle}{\left(\Delta_k^{\text{ret}}\right)^2} + \frac{\left(\left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret} \right\rangle - c^2\right)\left\langle \vec{u}^{(k)ret}, \dot{\vec{u}}^{(k)ret} \right\rangle}{\left(\Delta_k^{\text{ret}}\right)^4} \right)}{\left(c^2\tau_k^{\text{ret}} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle\right)^2} \right] \right\}$$

$$- \frac{e_k^2}{2m_k c^2} \left\{ \frac{ic\tau_k^{adv} \frac{\left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle - c^2}{\Delta_k \Delta_k^{adv}} - \frac{ic}{\Delta_k^{adv}} \frac{\left\langle \vec{u}^{(k)}, \vec{\xi}^{(k)adv} \right\rangle - c^2\tau_k^{adv}}{\Delta_k}}{\left(\frac{\left\langle \vec{u}^{(k)adv}, \vec{\xi}^{(k)adv} \right\rangle - c^2\tau_k^{adv}}{\Delta_k^{adv}}\right)^3} H_k^{adv} + \right.$$

$$+ \left(\Delta_k^{adv}\right)^2 \left[\frac{\left[\frac{\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)} \right\rangle - c^2\tau_k^{adv}}{\Delta_k} \frac{icD_k^{adv} \left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv} \right\rangle}{\left(\Delta_k^{adv}\right)^4} \right.}{\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle\right)^2} \right.$$

$$\left. \left. - \frac{ic\tau_k^{adv} \frac{D_k^{adv}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)adv} \right\rangle}{\left(\Delta_k^{adv}\right)^2} + \frac{\left(\left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle - c^2\right)\left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv} \right\rangle}{\left(\Delta_k^{adv}\right)^4} \right)}{\left(c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle\right)^2} \right] \right\}.$$

Remark 3.1. The above system contains 16 equations while the unknown functions are 12 in number. One can prove as in [2] that equations (5) are consequence of (4). Therefore, the system (4) contains exactly 12 equations which we consider in the following.

4. Transformation of the System of Equations

We simplify Eq. (4) using denotations:

$$A_{kn}=\frac{H_{kn}\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)}{\left(c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle\right)^3}-D_{kn}\frac{\Delta_{kn}^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(n)}\right\rangle+\left(\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle-c^2\right)\left\langle \vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};$$

$$B_{kn}=\frac{H_{kn}\left(c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_{kn}-\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle\right)^3}-\frac{D_{kn}\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-c^2\tau_{kn}\right)\left\langle \vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};C_{kn}=\frac{D_{kn}\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-c^2\tau_{kn}\right)}{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^2};$$

$$A_k^{ret}=\frac{H_k^{ret}\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)}{\left(c^2\tau_k^{ret}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}-D_k^{ret}\frac{\left(\Delta_k^{ret}\right)^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle+\left(\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle-c^2\right)\left\langle \vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_k^{\text{ret}}\right)^2\left(c^2\tau_k^{\text{ret}}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};$$

$$B_k^{ret}=\frac{H_k^{ret}\left(c^2\tau_k^{\text{ret}}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_k^{ret}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}-\frac{D_k^{ret}\left(\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{\text{ret}}\right)\left\langle \vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(\Delta_k^{ret}\right)^2\left(c^2\tau_k^{ret}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};C_k^{\text{ret}}=\frac{D_k^{ret}\left(\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{\text{ret}}\right)}{\left(c^2\tau_k^{\text{ret}}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^2};$$

$$A_k^{adv}=\frac{H_k^{adv}\left(c^2-\left\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)}{\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^3}-D_k^{adv}\frac{\left(\Delta_k^{adv}\right)^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle+\left(\left\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle-c^2\right)\left\langle \vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_k^{adv}\right)^2\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^2};$$

$$B_k^{adv}=\frac{H_k^{adv}\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}\right\rangle\right)}{\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^3}-\frac{D_k^{adv}\left(\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{adv}\right)\left\langle \vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(\Delta_k^{adv}\right)^2\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^2};C_k^{adv}=\frac{D_k^{adv}\left(\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}\right\rangle-c^2\tau_k^{adv}\right)}{\left(c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle\right)^2}$$

$$\begin{aligned}
&\dot{u}_\alpha^{(1)}(t)+\frac{u_\alpha^{(1)}(t)}{\Delta_1^2}\left\langle \vec{u}^{(1)}(t),\dot{\vec{u}}^{(1)}(t)\right\rangle=\\
&=\Delta_1\frac{e_1e_2\left(A_{12}\xi_\alpha^{(12)}-B_{12}u_\alpha^{(2)}+C_{12}\dot{u}_\alpha^{(2)}\right)+e_1e_3\left(A_{13}\xi_\alpha^{(13)}-B_{13}u_\alpha^{(3)}+C_{13}\dot{u}_\alpha^{(3)}\right)+e_1e_4\left(A_{14}\xi_\alpha^{(14)}-B_{14}u_\alpha^{(4)}+C_{14}\dot{u}_\alpha^{(4)}\right)}{m_1c^2}\\
&+\Delta_1\frac{e_1^2}{m_1c^2}\frac{A_1^{\text{ret}}\,\xi_\alpha^{(1)ret}-B_1^{\text{ret}}\,u_\alpha^{(1)}+C_1^{ret}u_\alpha^{(1)ret}-A_1^{adv}\xi_\alpha^{(1)adv}+B_1^{adv}u_\alpha^{(1)}-C_1^{adv}u_\alpha^{(1)adv}}{2};\\
&\dot{u}_\alpha^{(2)}(t)+\frac{u_\alpha^{(2)}(t)}{\Delta_2^2}\left\langle \vec{u}^{(2)}(t),\dot{\vec{u}}^{(2)}(t)\right\rangle\\
&=\Delta_2\frac{e_2e_1\left(A_{21}\xi_\alpha^{(21)}-B_{21}u_\alpha^{(1)}+C_{21}\dot{u}_\alpha^{(1)}\right)+e_2e_3\left(A_{23}\xi_\alpha^{(23)}-B_{23}u_\alpha^{(3)}+C_{23}\dot{u}_\alpha^{(3)}\right)+e_2e_4\left(A_{24}\xi_\alpha^{(24)}-B_{24}u_\alpha^{(4)}+C_{24}\dot{u}_\alpha^{(4)}\right)}{m_2c^2}\\
&+\Delta_2\frac{e_2^2}{m_2c^2}\frac{A_2^{\text{ret}}\,\xi_\alpha^{(2)ret}-B_2^{\text{ret}}\,u_\alpha^{(2)}+C_2^{ret}u_\alpha^{(2)ret}-A_2^{adv}\xi_\alpha^{(2)adv}+B_2^{adv}u_\alpha^{(2)}-C_2^{adv}u_\alpha^{(2)adv}}{2};\\
&\dot{u}_\alpha^{(3)}(t)+\frac{u_\alpha^{(3)}(t)}{\Delta_3^2}\left\langle \vec{u}^{(3)}(t),\dot{u}^{(3)}(t)\right\rangle\\
&=\Delta_3\frac{e_3e_1\left(A_{31}\xi_\alpha^{(31)}-B_{31}u_\alpha^{(1)}+C_{31}\dot{u}_\alpha^{(1)}\right)+e_3e_2\left(A_{32}\xi_\alpha^{(32)}-B_{32}u_\alpha^{(2)}+C_{32}\dot{u}_\alpha^{(2)}\right)+e_3e_4\left(A_{34}\xi_\alpha^{(34)}-B_{34}u_\alpha^{(4)}+C_{34}\dot{u}_\alpha^{(4)}\right)}{m_3c^2}\\
&+\Delta_3\frac{e_3^2}{m_3c^2}\frac{A_3^{ret}\xi_\alpha^{(3)ret}-B_3^{ret}u_\alpha^{(3)}+C_3^{ret}u_\alpha^{(3)ret}-A_3^{adv}\xi_\alpha^{(3)adv}+B_3^{adv}u_\alpha^{(3)}-C_3^{adv}u_\alpha^{(3)adv}}{2};\\
&\dot{u}_\alpha^{(4)}(t)+\frac{u_\alpha^{(4)}(t)}{\Delta_4^2}\left\langle \vec{u}^{(4)}(t),\dot{\vec{u}}^{(4)}(t)\right\rangle\\
&=\Delta_4\frac{e_4e_1\left(A_{41}\xi_\alpha^{(41)}-B_{41}1_\alpha^{(1)}+C_{41}\dot{u}_\alpha^{(1)}\right)+e_4e_2\left(A_{42}\xi_\alpha^{(42)}-B_{42}u_\alpha^{(2)}+C_{42}\dot{u}_\alpha^{(2)}\right)+e_4e_3\left(A_{43}\xi_\alpha^{(43)}-B_{43}u_\alpha^{(3)}+C_{43}\dot{u}_\alpha^{(3)}\right)}{m_4c^2}\\
&+\Delta_4\frac{e_4^2}{m_4c^2}\frac{A_4^{ret}\xi_\alpha^{(4)ret}-B_4^{ret}u_\alpha^{(4)}+C_4^{ret}u_\alpha^{(4)ret}-A_4^{adv}\xi_\alpha^{(4)adv}+B_4^{adv}u_\alpha^{(4)}-C_4^{adv}u_\alpha^{(4)adv}}{2}.
\end{aligned}$$

Recall our basic **Assumption (C)**: All velocities satisfy the inequalities

$$\left|u_\alpha{}^{(k)}(t)\right|\le\sqrt{\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}\le\bar{c}<c.$$

In view of Sommerfeld fine structure constant $\beta=\bar{c}/c\approx 1/137\Rightarrow\beta^2\approx 0$ we conclude that $\Delta_k\approx c$.

Then $(k=1,2,3,4;\alpha=1,2,3)$ for the Lorentz forces we obtain:

$$G_\alpha^{(12)}=\frac{\Delta_1 e_1 e_2\left(A_{12}\xi_\alpha^{(12)}-B_{12}u_\alpha^{(2)}+C_{12}\dot{u}_\alpha^{(2)}\right)}{m_1c^2}\approx\frac{e_1e_2\left(A_{12}\xi_\alpha^{(12)}-B_{12}u_\alpha^{(2)}+C_{12}\dot{u}_\alpha^{(2)}\right)}{m_1c};$$
$$G_\alpha^{(13)}=\frac{\Delta_1 e_1 e_3\left(A_{13}\xi_\alpha^{(13)}-B_{13}u_\alpha^{(3)}+C_{13}\dot{u}_\alpha^{(3)}\right)}{m_1c^2}\approx\frac{e_1e_3\left(A_{13}\xi_\alpha^{(13)}-B_{13}u_\alpha^{(3)}+C_{13}\dot{u}_\alpha^{(3)}\right)}{m_1c};$$
$$G_\alpha^{(14)}=\frac{\Delta_1 e_1 e_4\left(A_{14}\xi_\alpha^{(14)}-B_{14}u_\alpha^{(4)}+C_{14}\dot{u}_\alpha^{(4)}\right)}{m_1c^2}\approx\frac{e_1e_4\left(A_{14}\xi_\alpha^{(14)}-B_{14}u_\alpha^{(4)}+C_{14}\dot{u}_\alpha^{(4)}\right)}{m_1c};$$
$$G_\alpha^{(21)}=\frac{\Delta_2 e_2 e_1\left(A_{21}\xi_\alpha^{(21)}-B_{21}u_\alpha^{(1)}+C_{21}\dot{u}_\alpha^{(1)}\right)}{m_2c^2}\approx\frac{e_2e_1\left(A_{21}\xi_\alpha^{(21)}-B_{21}u_\alpha^{(1)}+C_{21}\dot{u}_\alpha^{(1)}\right)}{m_2c};$$
$$G_\alpha^{(23)}=\frac{\Delta_2 e_2 e_3\left(A_{23}\xi_\alpha^{(23)}-B_{23}u_\alpha^{(3)}+C_{23}\dot{u}_\alpha^{(3)}\right)}{m_2c^2}\approx\frac{e_2e_3\left(A_{23}\xi_\alpha^{(23)}-B_{23}u_\alpha^{(3)}+C_{23}\dot{u}_\alpha^{(3)}\right)}{m_2c};$$
$$G_\alpha^{(24)}=\frac{\Delta_2 e_2 e_4\left(A_{24}\xi_\alpha^{(24)}-B_{24}u_\alpha^{(4)}+C_{24}\dot{u}_\alpha^{(4)}\right)}{m_2c^2}\approx\frac{e_2e_4\left(A_{24}\xi_\alpha^{(24)}-B_{24}u_\alpha^{(4)}+C_{24}\dot{u}_\alpha^{(4)}\right)}{m_2c}$$
$$G_\alpha^{(31)}=\frac{\Delta_3 e_3 e_1\left(A_{31}\xi_\alpha^{(31)}-B_{31}u_\alpha^{(1)}+C_{32}\dot{u}_\alpha^{(31)}\right)}{m_3c^2}\approx\frac{e_3e_1\left(A_{31}\xi_\alpha^{(31)}-B_{31}u_\alpha^{(1)}+C_{32}\dot{u}_\alpha^{(31)}\right)}{m_3c};$$
$$G_\alpha^{(32)}=\frac{\Delta_3 e_3 e_2\left(A_{32}\xi_\alpha^{(32)}-B_{32}u_\alpha^{(2)}+C_{32}\dot{u}_\alpha^{(2)}\right)}{m_3c^2}\approx\frac{e_3e_2\left(A_{32}\xi_\alpha^{(32)}-B_{32}u_\alpha^{(2)}+C_{32}\dot{u}_\alpha^{(2)}\right)}{m_3c};$$
$$G_\alpha^{(34)}=\frac{\Delta_3 e_3 e_4\left(A_{34}\xi_\alpha^{(34)}-B_{34}u_\alpha^{(4)}+C_{34}\dot{u}_\alpha^{(4)}\right)}{m_3c^2}\approx\frac{e_3e_4\left(A_{34}\xi_\alpha^{(34)}-B_{34}u_\alpha^{(4)}+C_{34}\dot{u}_\alpha^{(4)}\right)}{m_3c};$$
$$G_\alpha^{(41)}=\frac{\Delta_4 e_4 e_1\left(A_{41}\xi_\alpha^{(41)}-B_{41}u_\alpha^{(1)}+C_{41}\dot{u}_\alpha^{(1)}\right)}{m_4c^2}\approx\frac{e_4e_1\left(A_{41}\xi_\alpha^{(41)}-B_{41}u_\alpha^{(1)}+C_{41}\dot{u}_\alpha^{(1)}\right)}{m_4c};$$
$$G_\alpha^{(32)}=\frac{\Delta_4 e_3 e_2\left(A_{42}\xi_\alpha^{(42)}-B_{42}u_\alpha^{(2)}+C_{42}\dot{u}_\alpha^{(2)}\right)}{m_4c^2}\approx\frac{e_3e_2\left(A_{42}\xi_\alpha^{(42)}-B_{42}u_\alpha^{(2)}+C_{42}\dot{u}_\alpha^{(2)}\right)}{m_4c};$$
$$G_\alpha^{(43)}=\frac{\Delta_4 e_4 e_3\left(A_{43}\xi_\alpha^{(43)}-B_{43}u_\alpha^{(4)}+C_{43}\dot{u}_\alpha^{(4)}\right)}{m_4c^2}\approx\frac{e_4e_3\left(A_{43}\xi_\alpha^{(43)}-B_{43}u_\alpha^{(4)}+C_{43}\dot{u}_\alpha^{(4)}\right)}{m_4c};$$
$$G_\alpha^{(k)rad}=\frac{\Delta_k e_k^2}{m_kc^2}\frac{A_k^{ret}\xi_\alpha^{(k)ret}-B_k^{ret}u_\alpha^{(k)}+C_k^{ret}u_\alpha^{(k)ret}-A_k^{adv}\xi_\alpha^{(k)adv}+B_k^{adv}u_\alpha^{(k)}-C_k^{adv}u_\alpha^{(k)adv}}{2}.$$

We write down the last system in the form

$$\dot{u}_\alpha^{(1)}+\frac{u_\alpha^{(1)}}{\Delta_1^2}\left\langle\vec{u}^{(1)},\dot{\vec{u}}^{(1)}\right\rangle=G_\alpha^{(12)}+G_\alpha^{(13)}+G_\alpha^{(14)}+G_\alpha^{(1)rad};$$
$$\dot{u}_\alpha^{(2)}+\frac{u_\alpha^{(2)}}{\Delta_2^2}\left\langle\vec{u}^{(2)},\dot{\vec{u}}^{(2)}\right\rangle=G_\alpha^{(21)}+G_\alpha^{(23)}+G_\alpha^{(24)}+G_\alpha^{(2)rad};$$
$$\dot{u}_\alpha^{(3)}+\frac{u_\alpha^{(3)}}{\Delta_3^2}\left\langle\vec{u}^{(3)},\dot{\vec{u}}^{(3)}\right\rangle=G_\alpha^{(31)}+G_\alpha^{(32)}+G_\alpha^{(34)}+G_\alpha^{(3)rad};$$
$$\dot{u}_\alpha^{(4)}+\frac{u_\alpha^{(4)}}{\Delta_4^2}\left\langle\vec{u}^{(4)},\dot{\vec{u}}^{(4)}\right\rangle=G_\alpha^{(31)}+G_\alpha^{(32)}+G_\alpha^{(34)}+G_\alpha^{(3)rad}.$$

For the rest of the equations we obtain

$$\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle}{\Delta_k^3} = \sum_{n=1,n\neq k}^{4} \frac{e_k e_n}{m_k c^2} \left\{ \frac{\left\langle \vec{\xi}^{(kn)}, \vec{u}^{(k)} \right\rangle - \tau_{kn} \left\langle \vec{u}^{(k)}, \vec{u}^{(n)} \right\rangle}{\left(\left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle - c^2 \tau_{kn} \right)^3} \Delta_{kn}^2 H_{kn} \right.$$
$$\left. + \frac{D_{kn} \left[\left(\left\langle \xi^{(kn)}, \vec{u}^{(k)} \right\rangle - \tau_{kn} \left\langle \vec{u}^{(k)}, \vec{u}^{(n)} \right\rangle \right) \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle - \tau_{kn} \Delta_{kn}^2 \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(n)} \right\rangle \right]}{\Delta_{kn}^2 \left(c^2 \tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle \right)^2} \right\}$$

$$+ \frac{e_k^2}{2m_k c^2} \left\{ \frac{\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)} \right\rangle - \tau_k^{\text{ret}} \left\langle \vec{u}^{(k)}, \vec{u}^{(kret} \right\rangle}{\left(\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle - c^2 \tau_k^{\text{ret}} \right)^3} \left(\Delta_k^{\text{ret}} \right)^2 H_k^{\text{ret}} \right.$$
$$\left. + \frac{D_k^{\text{ee}} \left[\left(\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)} \right\rangle - \tau_k^{\text{ret}} \left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret} \right\rangle \right) \left\langle \vec{u}^{(k)ret}, \dot{\vec{u}}^{(k)ret} \right\rangle - \tau_k^{\text{ret}} \left(\Delta_k^{\text{ret}} \right)^2 \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)ret} \right\rangle \right]}{\left(\Delta_k^{\text{ret}} \right)^2 \left(c^2 \tau_k^{\text{ret}} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle \right)^2} \right\}$$

$$+ \frac{e_k^2}{2m_k c^2} \left\{ \frac{\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)} \right\rangle - \tau_k^{adv} \left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle}{\left(\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle - c^2 \tau_k^{adv} \right)^3} \left(\Delta_k^{adv} \right)^2 H_k^{adv} \right.$$
$$\left. + \frac{D_k^{adv} \left[\left(\left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)} \right\rangle - \tau_k^{adv} \left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle \right) \left\langle \vec{u}^{(k)adv}, \dot{\vec{u}}^{(k)adv} \right\rangle - \tau_k^{adv} \left(\Delta_k^{adv} \right)^2 \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)adv} \right\rangle \right]}{\left(\Delta_k^{adv} \right)^2 \left(c^2 \tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle \right)^2} \right\}.$$

The assumption **(C)** implies $c^2 - \left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle \geq c^2 - \bar{c}^2 > 0$. Therefore, the determinant of the above system is $\delta = c^2/\Delta_k^2 > 0$ and consequently we can solve the last system with respect to $\dot{u}_\alpha^{(k)}(t), (k = 1, 2, 3, 4; \alpha = 1, 2, 3)$:

$$\dot{u}_1^{(1)}(t) = \frac{\left(c^2 - \left(u_1^{(1)}\right)^2\right) \left(G_1^{(12)} + G_1^{(13)} + G_1^{(14)} + G_1^{(1)rad}\right)}{c^2} - \frac{u_1^{(1)} u_2^{(1)} \left(G_2^{(12)} + G_2^{(13)} + G_2^{(14)} + G_2^{(2)rad}\right)}{c^2}$$
$$- \frac{u_1^{(1)} u_3^{(1)} \left(G_3^{(12)} + G_3^{(13)} + G_3^{(14)} + G_3^{(3)rad}\right)}{c^2} \equiv V_1^{(1)},$$
$$\dot{u}_2^{(1)}(t) = -\frac{u_1^{(1)} u_2^{(1)} \left(G_1^{(12)} + G_1^{(13)} + G_1^{(14)} + G_1^{(1)\text{rad}}\right)}{c^2} + \frac{\left(c^2 - \left(u_2^{(1)}\right)^2\right) \left(G_2^{(12)} + G_2^{(13)} + G_2^{(14)} + G_2^{(2)\text{rad}}\right)}{c^2}$$
$$- \frac{u_2^{(1)} u_3^{(1)} \left(G_3^{(12)} + G_3^{(13)} + G_3^{(14)} + G_3^{(3)\text{rad})}\right)}{c^2} \equiv U_2^{(1)};$$
$$\dot{u}_3^{(1)}(t) = -\frac{u_1^{(1)} u_3^{(1)} \left(G_1^{(12)} + G_1^{(13)} + G_1^{(14)} + G_1^{(1)rad}\right)}{c^2} - \frac{u_2^{(1)} u_3^{(1)} \left(G_2^{(12)} + G_2^{(13)} + G_2^{(14)} + G_2^{(2)rad}\right)}{c^2}$$
$$+ \frac{\left(c^2 - \left(u_3^{(1)}\right)^2\right) \left(G_3^{(12)} + G_3^{(13)} + G_3^{(14)} + G_3^{(3)rad}\right)}{c^2} \equiv U_3^{(1)};$$

$$\dot{u}_1^{(2)}(t) = \frac{\left(c^2 - \left(u_1^{(2)}\right)^2\right)\left(G_1^{(21)} + G_1^{(23)} + G_1^{(24)} + G_1^{(2)rad}\right)}{c^2} - \frac{u_1^{(2)}u_2^{(2)}\left(G_2^{(21)} + G_2^{(23)} + G_2^{(24)} + G_2^{(2)rad}\right)}{c^2}$$
$$- \frac{u_1^{(2)}u_3^{(2)}\left(G_3^{(21)} + G_3^{(23)} + G_3^{(24)} + G_3^{(2)rad}\right)}{c^2} \equiv U_1^{(2)};$$
$$\dot{u}_2^{(2)}(t) = -\frac{u_1^{(2)}u_2^{(2)}\left(G_1^{(21)} + G_1^{(23)} + G_1^{(24)} + G_1^{(2)rad}\right)}{c^2} + \frac{\left(c^2 - \left(u_2^{(2)}\right)^2\right)\left(G_2^{(21)} + G_2^{(23)} + G_2^{(24)} + G_2^{(2)rad}\right)}{c^2}$$
$$- \frac{u_2^{(2)}u_3^{(2)}\left(G_3^{(21)} + G_3^{(23)} + G_3^{(24)} + G_3^{(2)rad}\right)}{c^2} \equiv U_2^{(2)}$$
$$\dot{u}_3^{(2)}(t) = -\frac{u_1^{(2)}u_3^{(2)}\left(G_1^{(21)} + G_1^{(23)} + G_1^{(24)} + G_1^{(2)rad}\right)}{c^2} - \frac{u_2^{(2)}u_3^{(2)}\left(G_2^{(21)} + G_2^{(23)} + G_2^{(24)} + G_2^{(2)rad}\right)}{c^2}$$
$$+ \frac{\left(c^2 - \left(u_3^{(2)}\right)^2\right)\left(G_3^{(21)} + G_3^{(23)} + G_3^{(24)} + G_3^{(2)rad}\right)}{c^2} \equiv U_3^{(2)};$$

$$\dot{u}_1^{(3)}(t) = \frac{\left(c^2 - \left(u_1^{(3)}\right)^2\right)\left(G_1^{(31)} + G_1^{(32)} + G_1^{(34)} + G_1^{(3)rad}\right)}{c^2} - \frac{u_1^{(3)}u_2^{(3)}\left(G_2^{(31)} + G_2^{(32)} + G_2^{(34)} + G_2^{(3)rad}\right)}{c^2}$$
$$- \frac{u_1^{(3)}u_3^{(3)}\left(G_3^{(31)} + G_3^{(32)} + G_3^{(34)} + G_3^{(3)rad}\right)}{c^2} \equiv U_1^{(3)};$$
$$u_2^{(3)}(t) = -\frac{u_1^{(3)}u_2^{(3)}\left(G_1^{(31)} + G_1^{(32)} + G_1^{(34)} + G_1^{(3)rad}\right)}{c^2} + \frac{\left(c^2 - \left(u_2^{(3)}\right)^2\right)\left(G_2^{(31)} + G_2^{(32)} + G_2^{(34)} + G_2^{(3)rad}\right)}{c^2}$$
$$- \frac{u_2^{(3)}u_3^{(3)}\left(G_3^{(31)} + G_3^{(32)} + G_3^{(34)} + G_3^{(3)rad}\right)}{c^2} \equiv U_2^{(3)};$$
$$u_3^{(3)}(t) = -\frac{u_1^{(3)}u_3^{(3)}\left(G_1^{(31)} + G_1^{(32)} + G_1^{(34)} + G_1^{(3)rad}\right)}{c^2} - \frac{u_2^{(3)}u_3^{(3)}\left(G_2^{(31)} + G_2^{(32)} + G_2^{(34)} + G_2^{(3)rad}\right)}{c^2}$$
$$+ \frac{\left(c^2 - \left(u_3^{(3)}\right)^2\right)\left(G_3^{(31)} + G_3^{(32)} + G_3^{(34)} + G_3^{(3)rad}\right)}{c^2} \equiv U_3^{(3)}.;$$
$$\dot{u}_1^{(4)}(t) = \frac{\left(c^2 - \left(u_1^{(4)}\right)^2\right)\left(G_1^{(41)} + G_1^{(42)} + G_1^{(43)} + G_1^{(4)rad}\right)}{c^2} - \frac{u_1^{(4)}u_2^{(4)}\left(G_2^{(41)} + G_2^{(42)} + G_2^{(43)} + G_2^{(4)rad)}\right)}{c^2}$$
$$- \frac{u_1^{(4)}u_3^{(4)}\left(G_3^{(41)} + G_3^{(42)} + G_3^{(43)} + G_3^{(4)rad}\right)}{c^2} \equiv U_1^{(4)};$$
$$u_2^{(4)}(t) = -\frac{u_1^{(4)}u_2^{(4)}\left(G_1^{(41)} + G_1^{(42)} + G_1^{(43)} + G_1^{(4)rad}\right)}{c^2} + \frac{\left(c^2 - \left(u_2^{(4)}\right)^2\right)\left(G_2^{(41)} + G_2^{(42)} + G_2^{(43)} + G_2^{(4)rad}\right)}{c^2}$$
$$- \frac{u_2^{(4)}u_3^{(4)}\left(G_3^{(41)} + G_3^{(42)} + G_3^{(43)} + G_3^{(4)rad}\right)}{c^2} \equiv U_2^{(4)};$$
$$i_3^{(4)}(t) = -\frac{u_1^{(4)}u_3^{(4)}\left(G_1^{(41)} + G_1^{(42)} + G_1^{(43)} + G_1^{(4)rad}\right)}{c^2} - \frac{u_2^{(4)}u_3^{(4)}\left(G_2^{(41)} + G_2^{(42)} + G_2^{(43)} + G_2^{(4)rad}\right)}{c^2}$$
$$+ \frac{\left(c^2 - \left(u_3^{(4)}\right)^2\right)\left(G_3^{(41)} + G_3^{(42)} + G_3^{(43)} + G_3^{(4)rad}\right)}{c^2} \equiv U_3^{(4)}.$$

5. Simplifying the Equations of Motion and Derivation the Radiation Terms

Indeed, assumption **(C)** implies

$$\dot{u}_2^{(1)}(t) = -\beta^2\left(G_1^{(12)}+G_1^{(13)}+G_1^{(14)}+G_1^{(1)rad}\right)+G_2^{(12)}+G_2^{(13)}+G_2^{(14)}+G_2^{(2)rad}-\beta^2\left(G_3^{(12)}+G_3^{(13)}+G_3^{(14)}+G_3^{(3)rad}\right)\equiv U_2^{(1);}$$

$$\dot{u}_3^{(1)}(t) = -\beta^2\left(G_1^{(12)}+G_1^{(13)}+G_1^{(14)}+G_1^{(1)rad}\right)-\beta^2\left(G_2^{(12)}+G_2^{(13)}+G_2^{(14)}+G_2^{(2)rad}\right)+G_3^{(12)}+G_3^{(13)}+G_3^{(14)}+G_3^{(3)rad}\equiv U_3^{(1);}$$

$$\dot{u}_1^{(2)}(t) = G_1^{(21)}+G_1^{(23)}+G_1^{(24)}+G_1^{(2)rad}-\beta^2\left(G_2^{(21)}+G_2^{(23)}+G_2^{(24)}+G_2^{(2)rad}\right)-\beta^2\left(G_3^{(21)}+G_3^{(23)}+G_3^{(24)}+G_3^{(2)rad}\right)\equiv U_1^{(2);};$$

$$\dot{u}_2^{(2)}(t) = -\beta^2\left(G_1^{(21)}+G_1^{(23)}+G_1^{(24)}+G_1^{(2)rad}\right)+G_2^{(21)}+G_2^{(23)}+G_2^{(24)}+G_2^{(2)rad}-\beta^2\left(G_3^{(21)}+G_3^{(23)}+G_3^{(24)}+G_3^{(2)rad}\right)\equiv U_2^{(2)};$$

$$\dot{u}_3^{(2)}(t) = -\beta^2\left(G_1^{(21)}+G_1^{(23)}+G_1^{(24)}+G_1^{(2)rad}\right)-\beta^2\left(G_2^{(21)}+G_2^{(23)}+G_2^{(24)}+G_2^{(2)rad}\right)+G_3^{(21)}+G_3^{(23)}+G_3^{(24)}+G_3^{(2)rad}\equiv U_3^{(2)};$$

$$\dot{u}_1^{(3)}(t) = G_1^{(31)}+G_1^{(32)}+G_1^{(34)}+G_1^{(3)rad}-\beta^2\left(G_2^{(31)}+G_2^{(32)}+G_2^{(34)}+G_2^{(3)rad}\right)-\beta^2\left(G_3^{(31)}+G_3^{(32)}+G_3^{(34)}+G_3^{(3)rad}\right)\equiv U_1^{(3);};$$

$$\dot{u}_2^{(3)}(t) = -\beta^2\left(G_1^{(31)}+G_1^{(32)}+G_1^{(34)}+G_1^{(3)rad}\right)+G_2^{(31)}+G_2^{(32)}+G_2^{(34)}+G_2^{(3)rad}-\beta^2\left(G_3^{(31)}+G_3^{(32)}+G_3^{(34)}+G_3^{(3)rad}\right)\equiv U_2^{(3)};$$

$$\dot{u}_1^{(4)}(t) = G_1^{(41)}+G_1^{(42)}+G_1^{(43)}+G_1^{(4)rad}-\beta^2\left(G_2^{(41)}+G_2^{(42)}+G_2^{(43)}+G_2^{(4)rad}\right)-\beta^2\left(G_3^{(41)}+G_3^{(42)}+G_3^{(43)}+G_3^{(4)rad}\right)\equiv U_1^{(4);};$$

$$\dot{u}_2^{(4)}(t) = -\beta^2\left(G_1^{(41)}+G_1^{(42)}+G_1^{(43)}+G_1^{(4)rad}\right)+G_2^{(41)}+G_2^{(42)}+G_2^{(43)}+G_2^{(4)rad}-\beta^2\left(G_3^{(41)}+G_3^{(42)}+G_3^{(43)}+G_3^{(4)rad}\right)\equiv U_2^{(4);};$$

$$\dot{u}_3^{(4)}(t) = -\beta^2\left(G_1^{(41)}+G_1^{(42)}+G_1^{(43)}+G_1^{(4)rad}\right)-\beta^2\left(G_2^{(41)}+G_2^{(42)}+G_2^{(43)}+G_2^{(4)rad}\right)+G_3^{(41)}+G_3^{(42)}+G_3^{(43)}+G_3^{(4)rad}\equiv U_3^{(4)}.$$

To obtain a suitable form of the radiation terms we follow the Dirac assumption $\tau_k^{ret}=\tau_k^{adv}=\tau$ (τ is a small parameter). This assumption is motivated by the fact that $\tau=\tau_0\sqrt{1-\beta^2}\approx\tau_0, \left(\tau_0=r_e/c\approx 10^{-24}\text{sec}\right)$.

Remark 5.1. In fact, the Dirac assumption can be (strictly from mathematical point of view) formulated using the nonstandard analysis (cf. [9, 14, 17]). Indeed, in the special relativity there are no objects with finite size. Therefore, the electron size r_e should be considered as infinitely small (but not zero) and then τ should be infinitely small. Consequently, the expressions $\frac{u(t+\tau)-u(t-\tau)}{2\tau}$ are H. Schwartz derivative of the corresponding velocities. Recall that all $u_\alpha^{(k)}(t)$ are infinitely smooth functions [10, 11]. Then

$$u_\alpha^{(k)}(t+\tau)\approx u_\alpha^{(k)}(t); u_\alpha^{(k)}(t-\tau)\approx u_\alpha^{(k)}(t); u_\alpha^{(k)}(t)u_\alpha^{(k)}(t+\tau)\approx\left(u_\alpha^{(k)}(t)\right)^2; u_\alpha^{(k)}(t)u_\alpha^{(k)}(t-\tau)\approx\left(u_\alpha^{(k)}(t)\right)^2;$$

$$\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\approx\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle;\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\approx\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle;\left\langle\vec{u}^{(k)adv},\vec{u}^{(k)adv}\right\rangle$$
$$\approx\left\langle\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle;\left\langle\vec{u}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\approx\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle;$$

$$c^2\tau_k^{ret}-\left\langle\vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle=c^2\tau-\tau\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t-\tau)\right\rangle\approx\tau c^2\left(1-\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle}{c^2}\right)\approx\tau c^2;$$

$$c^2\tau_k^{adv}-\left\langle\vec{\xi}^{(k)adv},\vec{u}^{(k)adv}\right\rangle=c^2\tau-\tau\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t+\tau)\right\rangle\approx\tau c^2\left(1-\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle}{c^2}\right)\approx\tau c^2;$$

$$\xi^{(k)ret}=\left(x_1^{(k)}(t)-x_1^{(k)}\left(t-\tau_k^{ret}\right),x_2^{(k)}(t)-x_2^{(k)}\left(t-\tau_k^{ret}\right),x_3^{(k)}(t)-x_3^{(k)}\left(t-\tau_k^{ret}\right),ic\tau_k^{ret}\right)$$
$$\approx\left(u_1^{(k)}(.)\tau,u_2^{(k)}(.)\tau,u_3^{(k)}(.)\tau,ic\tau\right);$$

$$\xi^{(k)adv}=\left(x_1^{(k)}\left(t+\tau_k^{adv}\right)-x_1^{(k)}(t),x_2^{(k)}\left(t+\tau_k^{adv}\right)-x_2^{(k)}(t),x_3^{(k)}\left(t+\tau_k^{adv}\right)-x_3^{(k)}(t),ic\tau_k^{adv}\right)$$
$$\approx\left(u_1^{(k)}(.)\tau,u_2^{(k)}(.)\tau,u_3^{(k)}(.)\tau,ic\tau\right);$$

$$\Delta_k=\sqrt{c^2-\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle\approx c};$$

$$c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle \approx c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)} \right\rangle \approx c^2\tau - \tau\left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle = c^2\tau;$$
$$c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle \approx c^2\tau - \left\langle \vec{\xi}^{(k)}, \vec{u}^{(k)} \right\rangle \approx c^2\tau - \tau\left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle = c^2\tau;$$
$$c^2\tau_k^{ret} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle = c^2\tau - \tau\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t-\tau) \right\rangle \approx \tau\left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle\right);$$
$$c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle = c^2\tau - \tau\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t+\tau) \right\rangle \approx \tau\left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle\right)$$

Then the explicit form of the radiation term is:

$$\begin{aligned}
G_\alpha^{(k)rad} = \frac{e_k^2\Delta_k}{2m_kc^2}\Bigg[&\Bigg(\frac{\Delta^2_{(k)ret}\left(c^2 - \langle \vec{u}^{(k)}, \vec{u}^{(k)ret}\rangle\right)}{\left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle\right)^3} \\
&+ \frac{\left(c^2 - \langle \vec{u}^{(k)}, \vec{u}^{(k)ret}\rangle\right)\vec{\xi}^{(k)ret}, \dot{\vec{u}}^{(k)ret}\Big\rangle - \left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle\right)\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}\right\rangle\right)\left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle\right)^2}\Bigg)\xi_\alpha^{(k)ret} \\
&- \Bigg(\frac{\Delta^2_{(k)adv}\left(c^2 - \langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\rangle\right)}{\left(c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^3} \\
&+ \frac{\left(c^2 - \langle \vec{u}^{(k)}, \vec{u}^{(k)adv}\rangle\right)k\vec{\xi}^{(k)adv}, \dot{\vec{u}}^{(k)adv}\Big\rangle - \left(c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)}\right\rangle\right)\left(c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)^2}\Bigg)\xi_\alpha^{(k)adv} \\
&+ \left(\frac{\Delta^2_{(k)adv}\left(c^2\tau - \left\langle \vec{u}^{(k)}, \vec{\xi}^{(k)adv}\right\rangle\right) - \left(c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle\right)\left\langle \vec{\xi}^{(k)adv}, \dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2\tau - \left\langle \vec{u}^{(k)adv}, \vec{\xi}^{(k)adv}\right\rangle\right)^3}\right)u_\alpha^{(k)adv} \\
&- \left(\frac{\Delta^2_{(k)ret}\left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}\right\rangle\right) - \left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle\right)\left\langle \vec{\xi}^{(k)ret}, \dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle\right)^3}\right)u_\alpha^{(k)ret} \\
&- \frac{\dot{u}_\alpha^{(k)adv}}{c^2\tau - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv}\right\rangle} + \frac{c^2\tau - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret}\right\rangle}{\dot{u}^{(k)ret}}\Bigg]
\end{aligned}$$

$$\approx \frac{e_k^2}{2m_k c}\left[\left(\frac{c^4}{(c^2\tau)^3}+\frac{c^2\tau\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle-\tau c^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{\tau^3 c^2\left(c^2\right)^2}\right)\tau u_\alpha^{(k)}(t)\right.$$

$$-\left(\frac{c^4}{(c^2\tau)^3}+\frac{c^2\tau\left\langle u^{(k)},\dot{u}^{(k)adv}\right\rangle-\tau c^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\tau^3\left(c^2\right)\left(c^2\right)^2}\right)\tau u_\alpha^{(k)}(t)$$

$$\left.+\frac{c^4\tau-c^2\tau\left\langle \vec{\xi}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle}{\left(c^2\tau-\left\langle \vec{u}^{(k)adv},\vec{\xi}^{(k)adv}\right\rangle\right)^3}u_\alpha^{(k)adv}-\frac{c^4\tau-c^2\tau\left\langle \vec{\xi}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle}{\left(c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)ret}\right\rangle\right)^3}u_\alpha^{(k)ret}-\frac{\dot{u}_\alpha^{(k)adv}}{c^2\tau}+\frac{\dot{u}_\alpha^{(k)ret}}{c^2\tau}\right]$$

$$\approx \frac{e_k^2}{2m_k c}\left[\frac{c^4\tau-c^2\tau^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{(c^2\tau)^3}u_\alpha^{(k)}-\frac{c^4\tau-c^2\tau^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{(c^2\tau)^3}u_\alpha^{(k)}-\frac{\dot{u}_\alpha^{(k)adv}}{c^2\tau}+\frac{\dot{u}_\alpha^{(k)ret}}{c^2\tau}\right]$$

$$\approx -\frac{e_k^2}{m_k c^3}\frac{1}{c^2}\left[\frac{\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t+\tau)\right\rangle-\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{2\tau}u_\alpha^{(k)}+\frac{\dot{u}_\alpha^{(k)}(t+\tau)-\dot{u}_\alpha^{(k)}(t-\tau)}{2\tau}\right]$$

$$\approx -\frac{e_k^2}{m_k c^3}\left(\frac{\left\langle \vec{u}^{(k)},\ddot{\vec{u}}^{(k)}\right\rangle u_\alpha^{(k)}}{c^2}+\ddot{u}_\alpha^{(k)}\right).$$

Besides the radiation term for the energy equations is:

$$D_k^{ret}=\frac{c^2\tau_k^{\mathrm{ret}}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}(t-\tau)\right\rangle}{c^2\tau_k^{ret}-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}(t)\right\rangle}=\frac{c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}(t-\tau)\right\rangle}{c^2\tau-\left\langle \vec{\xi}^{(k)ret},\vec{u}^{(k)}(t)\right\rangle}\approx 1,$$

$$D_k^{adv}=\frac{c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}\left(t+\tau_k^{adv}\right)\right\rangle}{c^2\tau_k^{adv}-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}(t)\right\rangle}=\frac{c^2\tau-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}(t+\tau)\right\rangle}{c^2\tau-\left\langle \vec{\xi}^{(k)adv},\vec{u}^{(k)}(t)\right\rangle}\approx 1,$$

$$H_k^{ret}=1+\frac{\tau\left\langle u_\alpha^{(k)},\dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{\Delta_k^2}+\frac{\left(\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau c^2\right)\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{\Delta_k^4}\approx$$

$$\approx 1+\frac{\tau\left\langle u_\alpha^{(k)},\dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{c^2}-\frac{\tau\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{c^2}=1,$$

$$H_k^{adv}=1+\frac{\tau\left\langle u_\alpha^{(k)},\dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{{\Delta_k}^2}+\frac{\left(\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-c^2\tau\right)\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{\Delta_k^4}\approx$$

$$\approx 1+\frac{\tau\left\langle u_\alpha^{(k)},\dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{c^2}-\frac{\tau\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{c^2}=1,$$

$$R_4^{(k)rad}=\frac{e_k^2}{2m_k c^2}\left\{\frac{\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\left(c^2\tau-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^3}\Delta_k^2 H_k^{\mathrm{et}}-\frac{\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)adx}\right\rangle-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\left(c^2\tau-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)adi}\right\rangle\right)^3}\Delta_k^2 H_k^{\alpha^k}\right.$$

$$+\frac{\left(\left\langle \tau\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)\left\langle \vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle-\tau\Delta_k^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{{\Delta_k}^2\left(c^2\tau-\left\langle \tau\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^2}$$

$$\left.-\frac{\left(\left\langle \tau\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle \vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)\left\langle \vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle-\tau\Delta_k^2\left\langle \vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta_k^2\left(c^2\tau-\left\langle \tau\vec{u}^{(k)},\vec{u}^{(k)adv}\right\rangle\right)^2}\right\}$$

$$= \frac{e_k^2}{2m_k c^2}\left\{\frac{\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)ret}\right\rangle\right)^3}\Delta_k^2-\frac{\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)ck}\right\rangle\right.}\Delta_k^2\right.$$
$$+\frac{\left(\left\langle\tau\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)\left\langle\vec{u}^{(k)ret},\dot{\vec{u}}^{(k)ret}\right\rangle-\tau\Delta_k^2\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)ret}\right\rangle}{{\Delta_k}^2\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}$$
$$\left.-\frac{\left(\left\langle\tau\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)\left\langle\vec{u}^{(k)adv},\dot{\vec{u}}^{(k)adv}\right\rangle-\tau\Delta_k^2\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)adv}\right\rangle}{\Delta_k^2\left(c^2\tau-\tau\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle\right)^2}\right\}$$
$$=\frac{e_k^2}{m_k c^2}\frac{1}{\Delta_k^4}\left\langle\vec{u}^{(k)},\frac{\dot{\vec{u}}^{(k)}(t+\tau)-\dot{\vec{u}}^{(k)}(t-\tau)}{2\tau}\right\rangle\approx\frac{e_k^2}{m_k c^6}\left\langle\vec{u}^{(k)},\ddot{\vec{u}}^{(k)}\right\rangle.$$

Finally for the energy equations we obtain:

$$\frac{\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^3}=\frac{e_k^2\left\langle\vec{u}^{(k)},\ddot{\vec{u}}^{(k)}\right\rangle}{m_k c^6}+\sum_{n=1,n\neq k}^{4}\frac{e_k e_n}{m_k c^2}\left[\frac{\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle-\tau_{kn}\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle}{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-c^2\tau_{kn}\right)^3}\Delta_{kn}^2 H_{kn}\right.$$
$$\left.+\frac{\left(\left\langle\xi^{(kn)},\vec{u}^{(k)}\right\rangle-\tau_{kn}\left\langle\vec{u}^{(k)},\vec{u}^{(n)}\right\rangle\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle-\tau_{kn}\Delta_{kn}^2\left\langle\vec{u}^{(k)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2\left(c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle\right)^2}\right].$$

6. Conclusion

We have derived the system of equations of motion describing the 4-body electrodynamics problem. As in 2- and 3-body problems the equations of motion are nonlinear neutral type plus radiation terms containing third derivatives.In Section 4. we justify the derivation of the radiation terms via non-standard analysis. In the next paper we shall prove an existence-uniqueness result by means of suitable fixed point theorem . We shall prove stability of the Lithium atom and Hydrogen molecule, too.

References

[1] V. G. Angelov, *The N-body problem in classical electrodynamics*, Physics Essays, **6** (2), 1993, 204-211.

[2] V. G. Angelov, J. M. Soriano, *Uniqueness of escape trajectories for N-body problem of classical electrodynamics*, Mathematical Sciences Research Journal, **8** (6), 2004, 184-195, ISSN: 1537-5978.

[3] V. G. Angelov, *On the original Dirac equations with radiation term.* Libertas Mathematica (Texas), 31, 2011, 57-86. ISSN: 0278-5307 (print version); 2182-567X (online version).

[4] V. G. Angelov, Periodic solutions of the Dirac-Lorentz equation. International Journal of Theoretical and Mathematical Physics, **9**(5), 2019, 136-152.DOI: 10.5923/j.ijtmp.20190905.03.

[5] V. G. Angelov, *Two-body problem of classical electrodynamics with radiation terms – derivation of equations (I).* International Journal of Theoretical and Mathematical Physics, **5**(5), 2015, 119-135.

[6] V. G. Angelov, *Two-body problem of classical electrodynamics with radiation terms – periodis solutions (II).* International Journal of Theoretical and Mathematical Physics, **6**(1), 2016, 1-25.

[7] V. G. Angelov, *The electromagnetic three-body problem with radiation terms – derivation of equations of motion (I).* Results in Nonlinear Analysis, **3** (2), 2020, 45-58. (ISSN 2636-7556).

[8] V. G. Angelov, *The electromagnetic three-body problem with radiation terms – existence-uniqueness of periodic orbit (II).* Results in Nonlinear Analysis, **3** (3), 2020, 137-158. (ISSN 2636-7556).

[9] M. Davis, *Applied Nonstandard Analysis,* J. Wiley and Sons, New York, 1977.

[10] P. A. M. Dirac, *Classical theory of radiating electrons.* Proc.Roy.Soc. (London), A1, 1938, 148-169.

[11] R. D. Driver, *A two-body problem of classical electrodynamics; one-dimensional case,* Annals of Physics (N.Y.), 21, 1963, 122-142.

[12] R. D. Driver, *A neutral system with state-dependent delay,* J. Diff. Equations, **54** (1), 1984, 73-86.

[13] G. Herglotz, *Gottingen Nachrichten*, Math.-naturwissenschaften. KL, 1904, p. 549.

[14] W. A. J. Luxemburg, *Non-standard analysis,* Lectures on Robinson's Theory of Infinitesimals and Infinitely Large Numbers, Pasadena,1962, revised edition ,1964.

[15] W. Pauli, *Relativitaetstheorie.* Encyklopedie der Mathematischen Wissenschaften, Band. 5, Heft 4, Art. 19, 1921.

[16] R. W. Pohl, *Optik und Atomphysik,* Springer Verlag, 1963.

[17] A. Robinson, *Non-standard analysis,* Studies in Logic and Foundation of Mathematics, North-Holland, 1966.

[18] A. Sommerfeld, *Annalen der Physik,* Bd 33, 1910, p. 665.

[19] A. Sommerfeld, *Atomic Structure and Spectral Lines,* London, Mathuen and Co., 1934.

[20] J. L. Synge, *On the electromagnetic two-body problem,* Proc. Roy. Soc. (London), A177, 1940, 118-139.