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Common fixed point theorem for two commutative mappings involving iterates

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Abstract

We establish a common fixed point theorem for two commutative mappings in a real Hilbert space under a joint condition involving both mappings and their iterates. Our condition directly describes the interaction between the two mappings and generalizes a condition involving the two mappings separately. The main theorem guarantees the existence of a common fixed point under a boundedness assumption on the common orbit. As special cases, we obtain a previously known common fixed point theorem and, in particular, the common fixed point theorem for two commutative nonexpansive mappings. Examples illustrating the applicability of the theorem are also presented.

Keywords: Common fixed point, commutative mappings, nonexpansive mappings, Hilbert space
2010 MSC: 47H10, 47H09

1. Introduction

Throughout this paper, H denotes a real Hilbert space with an inner product $\langle \cdot, \ \cdot \rangle$ and the induced norm $\|\cdot\|$. Let C be a nonempty subset of H. We denote by $F(T) = \{x \in C : Tx = x\}$ the set of fixed points of T and by $\mathbb{N}$ the set of positive integers.

In 1965, Browder [4], Göhde [6], and Kirk [9] established fixed point theorems for nonexpansive mappings in Banach spaces. A nonexpansive mapping $T : C \to H$ is defined by the condition $\|Tx - Ty\| \leq \|x - y\|$ for

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all $x, y \in C$. Since then, the theory of nonexpansive mappings has been extensively developed and has found applications in various areas. For applications, see Blum and Oettli [3] for equilibrium problems, Byrne *et al.* [5] for split common fixed point problems, and Hao and Zhao [7] for split feasibility problems.

Several researchers have attempted to extend the class of mappings. In 2010, Kocourek *et al.* [10] defined a class of *generalized hybrid mappings* by the following condition:

$$\alpha \|Tx - Ty\|^2 + (1 - \alpha) \|x - Ty\|^2 \leq \beta \|Tx - y\|^2 + (1 - \beta) \|x - y\|^2 \tag{1}$$

for all $x, y \in C$, where $\alpha, \beta \in \mathbb{R}$. Nonexpansive mappings are included as the special case where $\alpha = 1$ and $\beta = 0$. The class of generalized hybrid mappings also contains nonspreading mappings [12], hybrid mappings [15], and λ-hybrid mappings [1]. Kocourek *et al.* [10] established a fixed point theorem, in addition to weak convergence theorems for finding a fixed point of a generalized hybrid mapping. For common fixed point theorems for generalized hybrid and more general types of mappings, see Hojo *et al.* [8].

In 2022, Kondo [13] proved the following common fixed point theorem:

Theorem 1.1 ([13])**.** *Let C be a nonempty, closed, and convex subset of H. Let S and T be mappings from C into itself such that $ST = TS$. Suppose that there exist $\lambda \in [0, 1]$ and $\alpha, \beta, \alpha', \beta' \in \mathbb{R}$ such that*

$$\begin{aligned}
& \lambda \left(\alpha \|Sx - Sy\|^2 + (1 - \alpha) \|x - Sy\|^2\right) \\
& + (1 - \lambda) \left(\alpha' \|Tx - Ty\|^2 + \left(1 - \alpha'\right) \|x - Ty\|^2\right) \\
\leq \; & \lambda \left(\beta \|Sx - y\|^2 + (1 - \beta) \|x - y\|^2\right) \\
& + (1 - \lambda) \left(\beta' \|Tx - y\|^2 + \left(1 - \beta'\right) \|x - y\|^2\right)
\end{aligned} \tag{2}$$

for all $x, y \in C$. Suppose that there exists $z \in C$ such that $\{S^k T^l z : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded. Then, $F(S)$ (resp. $F(T)$) is nonempty if $\lambda \in (0, 1]$ (resp. $\lambda \in [0, 1)$). In particular, if $\lambda \in (0, 1)$, then $F(S) \cap F(T)$ is nonempty.

The inequality (2) is a convex combination of (1) for S and T. If S and T are both generalized hybrid mappings, then (2) is satisfied. Thus, Theorem 1.1 includes a common fixed point theorem for generalized hybrid mappings as a special case.

Theorem 1.1 is distinguished from existing common fixed point theorems by the fact that it specifies a single condition (2) to be satisfied jointly by two mappings S and T. Kondo [13] also provided some examples of mappings that satisfy (2). For further development, see also Kondo [14].

Setting $\alpha = \alpha' = 1$ and $\beta = \beta' = 0$ in (2), we obtain the following corollary:

Corollary 1.2. *Let C be a nonempty, closed, and convex subset of H. Let S and T be mappings from C into itself such that $ST = TS$. Suppose that there exists $\lambda \in [0, 1]$ such that*

$$\lambda \|Sx - Sy\|^2 + (1 - \lambda) \|Tx - Ty\|^2 \leq \|x - y\|^2 \tag{3}$$

for all $x, y \in C$. Suppose that there exists $z \in C$ such that $\{S^k T^l z : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded. Then, $F(S)$ (resp. $F(T)$) is nonempty if $\lambda \in (0, 1]$ (resp. $\lambda \in [0, 1)$). In particular, if $\lambda \in (0, 1)$, then $F(S) \cap F(T)$ is nonempty.

If S and T are nonexpansive mappings, then condition (3) is satisfied. Therefore, we obtain a common fixed point theorem for two commutative nonexpansive mappings:

Corollary 1.3. *Let C be a nonempty, closed, and convex subset of H. Let S and T be nonexpansive mappings from C into itself such that $ST = TS$. Suppose that there exists $z \in C$ such that $\{S^k T^l z : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded. Then, $F(S) \cap F(T)$ is nonempty.*

Although condition (3) in Corollary 1.2 seems natural as a convex combination, it is somewhat restrictive. Indeed, for $\lambda \in (0,1)$, (3) implies that

$$\|Sx - Sy\|^2 \leq \frac{1}{\lambda} \|x - y\|^2, \tag{4}$$

since $(1-\lambda)\|Tx - Ty\|^2 \geq 0$. By (4), S is Lipschitz continuous. Similarly, T is also Lipschitz continuous. Thus, if S is not nonexpansive, then T must be a contraction. In this sense, condition (3) is a bit restrictive although Corollary 1.2 yields Corollary 1.3. Therefore, Theorem 1.1, which is a significant generalization of Corollary 1.2, is worthy of study.

In this paper, we establish a common fixed point theorem for two commutative mappings S and T under a condition that generalizes (3) in a direction different from (2). One distinctive feature is the incorporation of a direct interaction between S and T. More precisely, our main theorem assumes an inequality

$$\lambda \|Sx - S^pT^qy\|^2 + (1-\lambda)\|Tx - S^rT^sy\|^2 \leq \|x - y\|^2 \quad \text{for all } x, y \in C, \tag{5}$$

involving iterates of both S and T, where $\lambda \in (0,1)$ and $p, q, r, s \in \mathbb{N} \cup \{0\}$. In (5), the mappings S and T interact more closely than is implied by condition (2). Setting $p = s = 1$ and $q = r = 0$ in (5), we obtain condition (3). Thus, our main theorem yields Corollary 1.2 as a special case, and hence also the common fixed point theorem for two commutative nonexpansive mappings stated in Corollary 1.3. In the next section (Section 2), after preparing a lemma, we establish the main theorem of this study. In Section 3, we provide examples and remarks.

2. Main result

In this section, we establish the main theorem. To this end, we first present a lemma that will be used in the proof of the main theorem:

Lemma 2.1. *Let C be a nonempty and convex subset of a real Hilbert space H. Let S and T be mappings from C into itself. Suppose that there exists $z \in C$ such that $\{S^kT^lz : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded. For $p, q \in \mathbb{N} \cup \{0\}$, define*

$$A_n = \frac{1}{n^2} \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} S^kT^lz \in C \quad \textit{and} \quad A_n^{pq} = \frac{1}{n^2} \sum_{k=p}^{n-1+p} \sum_{l=q}^{n-1+q} S^kT^lz \in C.$$

Then, $A_n - A_n^{pq} \to 0$ as $n \to \infty$.

Proof. For simplicity, we give the proof for the case where $p = 1$ and $q = 0$. Since $\{S^kT^lz : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded,

$$\exists M > 0 \text{ such that } \forall k, l \in \mathbb{N} \cup \{0\}, \quad \left\|S^kT^lz\right\| \leq M. \tag{6}$$

Note that the following holds:

$$\begin{aligned}
& \left\|A_n - A_n^{10}\right\| \\
= & \left\| \frac{1}{n^2} \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} S^k T^l z - \frac{1}{n^2} \sum_{k=1}^{n} \sum_{l=0}^{n-1} S^k T^l z \right\| \\
= & \frac{1}{n^2} \left\| \sum_{k=0}^{n-1} \left(S^k z + S^k T z + \cdots + S^k T^{n-1} z \right) - \sum_{k=1}^{n} \left(S^k z + S^k T z + \cdots + S^k T^{n-1} z \right) \right\| \\
= & \frac{1}{n^2} || \left(z + Tz + \cdots + T^{n-1} z \right) + \left(Sz + STz + \cdots + ST^{n-1} z \right) \\
& + \cdots\cdots \\
& + \left(S^{n-1} z + S^{n-1} T z + \cdots + S^{n-1} T^{n-1} z \right) \\
& - \left(Sz + STz + \cdots + ST^{n-1} z \right) - \left(S^2 z + S^2 T z + \cdots + S^2 T^{n-1} z \right) \\
& - \cdots\cdots \\
& - \left(S^{n-1} z + S^{n-1} T z + \cdots + S^{n-1} T^{n-1} z \right) - \left(S^n z + S^n T z + \cdots + S^n T^{n-1} z \right) || \\
= & \frac{1}{n^2} \left\| \left(z + Tz + \cdots + T^{n-1} z \right) - \left(S^n z + S^n T z + \cdots + S^n T^{n-1} z \right) \right\| .
\end{aligned}$$

From (6), we obtain

$$\left\|A_n - A_n^{10}\right\| \le \frac{1}{n^2} \left(nM + nM\right) = \frac{2M}{n} \to 0$$

as $n \to \infty$.

The case for general $p, q \in \mathbb{N} \cup \{0\}$ can be proved similarly. This completes the proof. □

We are now in a position to prove the main theorem.

Theorem 2.2. *Let C be a nonempty, closed, and convex subset of a real Hilbert space H. Let S and T be mappings from C into itself such that $ST = TS$. Suppose that there exist $\lambda \in [0, 1]$ and $p, q, r, s \in \mathbb{N} \cup \{0\}$ such that*

$$\lambda \left\|Sx - S^p T^q y\right\|^2 + (1 - \lambda) \left\|Tx - S^r T^s y\right\|^2 \le \|x - y\|^2 \quad \text{for all } x, y \in C. \tag{7}$$

Suppose that there exists $z \in C$ such that $\{S^k T^l z : k, l \in \mathbb{N} \cup \{0\}\}$ is bounded. Then, $F(S)$ (resp. $F(T)$) is nonempty if $\lambda \in (0, 1]$ (resp. $\lambda \in [0, 1)$). In particular, if $\lambda \in (0, 1)$, then $F(S) \cap F(T)$ is nonempty.

Proof. For simplicity, we prove for the case where $p = 1$, $q = 2$, $r = 2$, and $s = 0$. Define

$$\begin{aligned}
A_n &= \frac{1}{n^2} \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} S^k T^l z \in C, \\
A_n^{12} &= \frac{1}{n^2} \sum_{k=1}^{n} \sum_{l=2}^{n+1} S^k T^l z \in C, \\
A_n^{20} &= \frac{1}{n^2} \sum_{k=2}^{n+1} \sum_{l=0}^{n-1} S^k T^l z \in C.
\end{aligned}$$

Since $\{A_n\}$ is bounded, there exist a subsequence $\{A_{n_i}\}$ and an element $v \in H$ such that $A_{n_i} \rightharpoonup v$. By Lemma 2.1, $A_n - A_n^{12} \to 0$ and $A_n - A_n^{20} \to 0$. Consequently, we have

$$A_n^{12} \rightharpoonup v \quad \text{and} \quad A_n^{20} \rightharpoonup v. \tag{8}$$

Since $\{A_n\} \subset C$ and C is weakly closed, it follows that $v \in C$. Since $S, T : C \to C$, we have $Sv, Tv \in C$. We aim to prove that $Sv = Tv = v$.

Let $k, l \in \mathbb{N} \cup \{0\}$. Using condition (7) for $x = v$, $y = S^k T^l z$, $p = 1$, $q = 2$, $r = 2$, and $s = 0$, together with the commutativity $ST = TS$, we obtain

$$\lambda \left\| Sv - S^{k+1}T^{l+2}z \right\|^2 + (1-\lambda) \left\| Tv - S^{k+2}T^l z \right\|^2 \leq \left\| v - S^k T^l z \right\|^2 .$$

From this,

$$\begin{aligned} &\lambda \left(\left\| S^{k+1}T^{l+2}z - v \right\|^2 + 2 \left\langle S^{k+1}T^{l+2}z - v,\ v - Sv \right\rangle + \|v - Sv\|^2 \right) \\ &+ (1-\lambda) \left(\left\| S^{k+2}T^l z - v \right\|^2 + 2 \left\langle S^{k+2}T^l z - v,\ v - Tv \right\rangle + \|v - Tv\|^2 \right) \\ &\leq \left\| S^k T^l z - v \right\|^2 . \end{aligned}$$

Therefore,

$$\begin{gathered} \lambda \left(\left\| S^{k+1}T^{l+2}z - v \right\|^2 - \left\| S^k T^l z - v \right\|^2 \right) \\ + \lambda \left(2 \left\langle S^{k+1}T^{l+2}z - v,\ v - Sv \right\rangle + \|v - Sv\|^2 \right) \\ + (1-\lambda) \left(\left\| S^{k+2}T^l z - v \right\|^2 - \left\| S^k T^l z - v \right\|^2 \right) \\ + (1-\lambda) \left(2 \left\langle S^{k+2}T^l z - v,\ v - Tv \right\rangle + \|v - Tv\|^2 \right) \leq 0. \end{gathered}$$

Summing these inequalities with respect to $k = 0, 1, \cdots, n-1$, we have

$$\begin{gathered} \lambda \left(\left\| S^n T^{l+2}z - v \right\|^2 - \left\| T^l z - v \right\|^2 \right) \\ + \lambda \left(2 \left\langle \sum_{k=1}^{n} S^k T^{l+2}z - nv,\ v - Sv \right\rangle + n \|v - Sv\|^2 \right) \\ + (1-\lambda) \left(\left\| S^{n+1}T^l z - v \right\|^2 + \left\| S^n T^l z - v \right\|^2 - \left\| S T^l z - v \right\|^2 - \left\| T^l z - v \right\|^2 \right) \\ + (1-\lambda) \left(2 \left\langle \sum_{k=2}^{n+1} S^k T^l z - nv,\ v - Tv \right\rangle + n \|v - Tv\|^2 \right) \leq 0. \end{gathered}$$

Summing these inequalities with respect to $l = 0, 1, \cdots, n-1$, we have

$$\begin{gathered} \lambda \left(\sum_{l=2}^{n+1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) \\ + \lambda \left(2 \left\langle \sum_{l=2}^{n+1} \sum_{k=1}^{n} S^k T^l z - n^2 v,\ v - Sv \right\rangle + n^2 \|v - Sv\|^2 \right) \\ + (1-\lambda) \left(\sum_{l=0}^{n-1} \left\| S^{n+1}T^l z - v \right\|^2 + \sum_{l=0}^{n-1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| S T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) \\ + (1-\lambda) \left(2 \left\langle \sum_{l=0}^{n-1} \sum_{k=2}^{n+1} S^k T^l z - n^2 v,\ v - Tv \right\rangle + n^2 \|v - Tv\|^2 \right) \leq 0. \end{gathered}$$

Since $ST = TS$, it follows that

$$\begin{aligned}
&\lambda \left(\sum_{l=2}^{n+1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) \\
&+\lambda \left(2 \left\langle \sum_{k=1}^{n} \sum_{l=2}^{n+1} S^k T^l z - n^2 v,\ v - Sv \right\rangle + n^2 \left\| v - Sv \right\|^2 \right) \\
&+ (1-\lambda) \left(\sum_{l=0}^{n-1} \left\| S^{n+1} T^l z - v \right\|^2 + \sum_{l=0}^{n-1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| S T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) \\
&+ (1-\lambda) \left(2 \left\langle \sum_{k=2}^{n+1} \sum_{l=0}^{n-1} S^k T^l z - n^2 v,\ v - Tv \right\rangle + n^2 \left\| v - Tv \right\|^2 \right) \le 0.
\end{aligned}$$

Dividing by n^2, we obtain

$$\begin{aligned}
&\frac{\lambda}{n^2} \left(\sum_{l=2}^{n+1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) + \lambda \left(2 \left\langle A_n^{12} - v,\ v - Sv \right\rangle + \left\| v - Sv \right\|^2 \right) \\
&+\frac{1-\lambda}{n^2} \left(\sum_{l=0}^{n-1} \left\| S^{n+1} T^l z - v \right\|^2 + \sum_{l=0}^{n-1} \left\| S^n T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| S T^l z - v \right\|^2 - \sum_{l=0}^{n-1} \left\| T^l z - v \right\|^2 \right) \\
&+ (1-\lambda) \left(2 \left\langle A_n^{20} - v,\ v - Tv \right\rangle + \left\| v - Tv \right\|^2 \right) \le 0
\end{aligned}$$

for all $n \in \mathbb{N}$. Replacing n by n_i and taking the limit as $i \to \infty$, we have from (8) that

$$\lambda \left\| v - Sv \right\|^2 + (1-\lambda) \left\| v - Tv \right\|^2 \le 0.$$

Thus, the desired results follow.

The case for general $p, q, r, s \in \mathbb{N} \cup \{0\}$ can be proved similarly by using

$$A_n^{pq} = \frac{1}{n^2} \sum_{k=p}^{n-1+p} \sum_{l=q}^{n-1+q} S^k T^l z \in C \quad \text{and} \quad A_n^{rs} = \frac{1}{n^2} \sum_{k=r}^{n-1+r} \sum_{l=s}^{n-1+s} S^k T^l z \in C.$$

This completes the proof. □

As stated in the Introduction, setting $p = s = 1$ and $q = r = 0$ in (7), we obtain Corollary 1.2. Thus, Corollary 1.3 also follows. In other words, commutative nonexpansive mappings have a common fixed point under condition (7) and the boundedness assumption on $\{S^k T^l z\}$. We have the following relationship among the results:

$$\begin{aligned}
&\text{Corollary 1.3} \subset \text{Corollary 1.2} \subset \text{Theorem 1.1}; \\
&\text{Corollary 1.3} \subset \text{Corollary 1.2} \subset \text{Theorem 2.2}.
\end{aligned} \tag{9}$$

This shows that Theorem 2.2 is a generalization of Corollaries 1.2 and 1.3 that differs from Theorem 1.1.

3. Examples and remarks

In this section, we provide examples that illustrate Theorem 2.2. Two remarks are also given.

3.1. Examples

As shown in (9), our main theorem (Theorem 2.2) contains the common fixed point theorem for commutative nonexpansive mappings (Corollary 1.3) as the special case where $p = s = 1$ and $q = r = 0$. We provide an example of commutative nonexpansive mappings. Let H be a real Hilbert space and let $B_r = \{x \in H : \|x\| \le r\}$ be a closed ball, where $r > 0$. Let S be a linear and isometric mapping defined on H, where "isometric" means that $\|Sx - Sy\| = \|x - y\|$ for all $x, y \in H$. Let $T = P_{B_r}$ be the metric projection from H onto B_r. Then, S and T are both nonexpansive and satisfy $ST = TS$. For s proof of the commutativity $ST = TS$, see Kohsaka [11] and Kondo [14] (Claim 4.2).

Next, we present an example in which S is not nonexpansive. Setting $\lambda = 1/16$, $p = r = 1$, and $q = 0$ in (7), we have

$$\frac{1}{16}\|Sx - Sy\|^2 + \frac{15}{16}\|Tx - ST^s y\|^2 \le \|x - y\|^2 \quad \text{for all } x, y \in C, \tag{10}$$

where $s = 1, 2, \cdots$. We present an example that satisfies (10). The example of a mapping is presented by Berinde [2]. Let $H = \mathbb{R}$ and let $C = \left[\frac{1}{2},\ 2\right]$. Define two mappings S and T as follows:

$$\begin{aligned} Sx &= \frac{1}{x} \quad \text{for all } x \in C; \\ Tx &= 1 \quad \text{for all } x \in C. \end{aligned} \tag{11}$$

Then, S and T are self-mappings defined on C. The mapping S is not nonexpansive, and S and T are commutative (i.e., $ST = TS$). The mappings S and T satisfy (10). Indeed, by (11), it follows that $ST^s y = 1$ for $s = 1, 2, \cdots$. Therefore,

$$\begin{aligned} &\frac{1}{16}\|Sx - Sy\|^2 + \frac{15}{16}\|Tx - ST^s y\|^2 - \|x - y\|^2 \\ &= \frac{1}{16}\left(\frac{1}{x} - \frac{1}{y}\right)^2 - (x - y)^2 \\ &= \frac{1}{16}\frac{(x - y)^2}{x^2 y^2} - (x - y)^2 \\ &= (x - y)^2\left(\frac{1}{16x^2 y^2} - 1\right) \le 0. \end{aligned}$$

This shows that (10) holds. In this case, $F(S) \cap F(T) = \{1\} \ne \emptyset$.

3.2. Remarks

We present two remarks. Theorem 2.2 holds for $p = q = r = s = 0$ and $\lambda \in (0, 1)$. In this case, condition (7) takes the following form:

$$\lambda\|Sx - y\|^2 + (1 - \lambda)\|Tx - y\|^2 \le \|x - y\|^2 \quad \text{for all } x, y \in C. \tag{12}$$

Since $(1 - \lambda)\|Tx - y\|^2 \ge 0$, it follows that

$$\lambda\|Sx - y\|^2 \le \|x - y\|^2 \quad \text{for all } x, y \in C.$$

Setting $x = y$, we obtain $Sx = x$. This means that S is the identity mapping. Similarly, $Tx = x$ must hold. Thus, in the case of (12), the main theorem becomes trivial.

Next, setting $q = r = 1$ and $p = s = 0$ in (7), we have

$$\lambda\|Sx - Ty\|^2 + (1 - \lambda)\|Tx - Sy\|^2 \le \|x - y\|^2 \quad \text{for all } x, y \in C. \tag{13}$$

If we set $x = y$ in (13), then $Sx = Tx$ for all $x \in C$, meaning that $S = T$. In this case, (13) is equivalent to the condition for a single nonexpansive mapping.

These remarks concerning (12) and (13) imply the following: when applying Theorem 2.2, it is necessary to carefully select parameters and examples to ensure non-triviality.

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Conflict of interest. The authors have no conflicts of interest.

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