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Spin 3-Body Problem Of Classical Electrodynamics In The 3D-Kepler Form

Vasil G. Angelov[a,*]

[a] *University of Mining and Geology "St. I. Rilski", Department of Mathematics and Informatics, 1700 Sofia, Bulgaria*

Abstract

In the present paper we prove the existence of spin functions for 3-body problem of classical electrodynamics. It is a direct continuation of a previous paper in which we proved the existence and uniqueness of a periodic solution to the same problem in 3D Kepler form. To prove the existence of periodic spin functions we use fixed point method for operator equations.

Keywords: spin functions, classical electrodynamics, three-body problem, periodic solutions, fixed point method
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1. Introduction

In [1, 2, 3] we have derived equations of motion for 3 charged particles in an internal for the particles frame of reference and proved the existence of periodic solutions. As a consequence, we have obtained periodic solutions for spin equations. The considerations are in a frame of reference internal for the moving particles. The spin equations are closely related to the equations of motion because the coefficients of the spin equations contain known already functions from the equations of motion for 3-body problem. In [4] we have considered 3-body problem in the 3D-Kepler formulation and proved the existence of periodic solution.

*Corresponding author
Email address: `angelov@mgu.bg` (Vasil G. Angelov)

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The main purpose of the present paper is to prove the existence of solutions for the spin equations for 3D-Kepler 3-body problem of classical electrodynamics using the results from [4].

The paper consists of five sections. In Section 2 we recall basic results for 3-body problem equations of motion with radiation terms and their relations to spin equations from [3, 4]. In Section 3 we obtain equations of motion in spherical coordinates and estimate the right-hand sides of the spin equations. In Section 4 we estimates of the right-hand sides of the spin equations. Section 5 contains the main result, namely, an existence of the spin functions. In Conclusion we point out that spin equations may have solutions with opposite signs.

2. Three-Body Problem Equations of Motion with Radiation Terms and Spin Equations in the Minkowski space

The general system describing the motion of three mass charged particles with radiation terms and spins in the frame of classical electrodynamics has been derived in [1]:

$$\begin{aligned}
\frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{m_1c^2}\left(\left(F_{rs}^{(12)}+F_{rs}^{(13)}\right)\lambda_s^{(1)}+F_{rs}^{(1)rad}\lambda_s^{(1)}\right);\\
\frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{m_2c^2}\left(\left(F_{rs}^{(21)}+F_{rs}^{(23)}\right)\lambda_s^{(2)}+F_{rs}^{(2)rad}\lambda_s^{(2)}\right);\\
\frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{m_3c^2}\left(\left(F_{rs}^{(31)}+F_{rs}^{(32)}\right)\lambda_s^{(3)}+F_{rs}^{(3)rad}\lambda_s^{(3)}\right);
\end{aligned} \tag{1}$$

$$\begin{aligned}
\frac{d\sigma_{ij}^{(1)}}{ds_1} &= \frac{e_1}{m_1c^2}\left(\left(F_{im}^{(12)}+F_{im}^{(13)}+F_{im}^{(1)rad}\right)\sigma_{mj}^{(1)}-\sigma_{im}^{(1)}\left(F_{mj}^{(12)}+F_{mj}^{(13)}+F_{mj}^{(1)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(2)}}{ds_2} &= \frac{e_2}{m_2c^2}\left(\left(F_{im}^{(21)}+F_{im}^{(23)}+F_{im}^{(2)rad}\right)\sigma_{mj}^{(2)}-\sigma_{im}^{(2)}\left(F_{mj}^{(21)}+F_{mj}^{(23)}+F_{mj}^{(2)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(3)}}{ds_3} &= \frac{e_3}{m_3c^2}\left(\left(F_{im}^{(31)}+F_{im}^{(32)}+F_{im}^{(3)rad}\right)\sigma_{mj}^{(3)}-\sigma_{im}^{(3)}\left(F_{mj}^{(31)}+F_{mj}^{(32)}+F_{mj}^{(3)rad}\right)\right).
\end{aligned} \tag{2}$$

The Roman subscribes run over $1,2,3,4$, while the Greek – over $1,2,3$ with Einstein summation convention. By $\langle .,. \rangle_4$ we denote the dot product in the Minkowski space, while by$\langle .,. \rangle$ the dot product in 3-dimensional Euclidean subspace.

The quantities relating to the particles are:

$$\left(x_1^{(k)}(t),x_2^{(k)}(t),x_3^{(k)}(t),x_4^{(k)}=ict\right)\equiv\left(\vec{x}^{(k)},ict\right)$$

space-time coordinates of the moving particles $P_k(k=1,2,3)$; c- the vacuum speed of light; m_k-proper masses; e_k- charges $(k=1,2,3)$. The elements of the electromagnetic tensors

$$F_{rl}^{(kn)}=\frac{\partial\Phi_l^{(n)}}{\partial x_r^{(k)}}-\frac{\partial\Phi_r^{(n)}}{\partial x_l^{(k)}}$$

can be calculated by the Lienard-Wiechert retarded potentials

$$\Phi_r^{(n)}=-\frac{e_n\lambda_r^{(n)}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4}$$

(cf. [5, 6, 7, 8]), where

$$\begin{aligned}
\xi^{(kn)} &= \left(\xi_1^{(kn)},\xi_2^{(kn)},\xi_3^{(kn)},\xi_4^{(kn)}\right)\\
&= \left(x_1^{(k)}(t)-x_1^{(n)}(t-\tau_{kn}),x_2^{(k)}(t)-x_2^{(n)}(t-\tau_{kn}),x_3^{(k)}(t)-x_3^{(n)}(t-\tau_{kn}),ic\tau_{kn}(t)\right)
\end{aligned}$$

are isotropic vectors and

$$\lambda^{(k)} = \left(\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}, \lambda_4^{(k)}\right) = \left(\vec{\lambda}^{(k)}, \lambda_4^{(k)}\right) = \left(\frac{u_1^{(k)}}{\Delta_k}, \frac{u_2^{(k)}}{\Delta_k}, \frac{u_3^{(k)}}{\Delta_k}, \frac{ic}{\Delta_k}\right) = \left(\frac{\vec{u}^{(k)}}{\Delta_k}, \frac{ic}{\Delta_k}\right);$$
$$\lambda^{(n)} = \left(\lambda_1^{(n)}, \lambda_2^{(n)}, \lambda_3^{(n)}, \lambda_4^{(n)}\right) = \left(\vec{\lambda}^{(n)}, \lambda_4^{(k)}\right) = \left(\frac{u_1^{(n)}}{\Delta_{kn}}, \frac{u_2^{(n)}}{\Delta_{kn}}, \frac{u_3^{(n)}}{\Delta_{kn}}, \frac{ic}{\Delta_{kn}}\right) = \left(\frac{\vec{u}^{(n)}}{\Delta_{kn}}, \frac{ic}{\Delta_{kn}}\right)$$

are unit tangent vectors to the world lines, where

$$\Delta_k = \sqrt{c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t)\right\rangle}; \Delta_{kn} = \sqrt{c^2 - \left\langle \vec{u}^{(n)}(t-\tau_{kn}), \vec{u}^{(n)}(t-\tau_{kn})\right\rangle}.$$

Since $\xi^{(kn)}$ are isotropic 4-vectors, i.e.

$$\left\langle \xi^{(kn)}, \xi^{(kn)} \right\rangle_4 = 0,$$

the retarded functions $\tau_{kn}(t)$ can be defined as solutions of the functional equations

$$\tau_{kn}(t) = \frac{1}{c}\sqrt{\left\langle \vec{\xi}^{(knn)}, \vec{\xi}^{(knn)} \right\rangle} = \frac{1}{c}\sqrt{\sum_{\alpha=1}^{3}\left[x_\alpha^{(k)}(t) - x_\alpha^{(n)}\left(t - \tau_{kn}(t)\right]^2\right.}$$

where $(kn) = (12), (13), (21), (23), (31), (32)$.

In [4] it is proved an existence-uniqueness of periodic solution of (1) under assumption

$$(C): \sqrt{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t)\right\rangle} \le \bar{c} < c(k = 1, 2, 3).$$

The system (1) in spherical coordinates becomes (cf. [4])

$$\begin{aligned}
\dot{r}_2 =& \frac{e_2 e_1}{m_2}\frac{1}{c{\rho_2}^2} - \frac{e_2 e_3}{m_2}\frac{\rho_2 - \rho_3\left(\cos\varphi_{23}\cos\lambda_2\cos\lambda_3 + \sin\lambda_2\sin\lambda_3\right)}{c^3}\frac{\left\langle \vec{\xi}^{(23)}, \dot{\vec{u}}^{(3)}\right\rangle}{c^3{\tau_{23}}^3}\\
&- \frac{e_2 e_3}{m_2}\frac{\dot{r}_3\left(\cos\varphi_{23}\cos\lambda_2\cos\lambda_3 + \sin\lambda_2\sin\lambda_3\right) + \rho_3\dot{\phi}_3\sin\varphi_{23}\cos\lambda_2\cos\lambda_3 + \rho_3\dot{\eta}_3\left(\sin\lambda_2\cos\lambda_3 - \cos\varphi_{23}\sin\lambda_3\cos\lambda_2\right)}{c^4\tau_{23}}\\
&- \frac{e_2^2}{m_2}\frac{\ddot{r}_2}{c^3} \equiv G_{r_2};
\end{aligned}$$

$$\dot{\phi}_2 = \frac{e_2 e_3}{m_2}\left[\frac{\rho_3\sin\varphi_{23}\cos\lambda_3}{{\tau_{23}}^3\rho_2\cos\lambda_2}\frac{\left\langle \vec{\xi}^{(23)}, \dot{\vec{u}}^{(3)}\right\rangle}{c^6} + \frac{\dot{r}_3\cos\lambda_3\sin\varphi_{23} + \rho_3\dot{\phi}_3\cos s_3\cos\varphi_{23} - \rho_3\dot{\eta}_3\sin\lambda_3\sin\varphi_{23}}{c^4\tau_{23}\rho_2\cos\lambda_2} - \frac{\ddot{\phi}_2}{c^3}\right] \equiv G_{\phi_2};$$

$$\begin{aligned}
\dot{\eta}_2 =& \frac{e_2 e_3}{m_2}\frac{\rho_3\cos\varphi_{23}\sin\lambda_2\cos\lambda_3}{c^3{\tau_{23}}^3}\frac{\left\langle \vec{\xi}^{(23)}, \dot{\vec{u}}^{(3)}\right\rangle}{c^3\rho_2}\\
&+ \frac{e_2 e_3}{m_2}\frac{\dot{r}_3\left(\cos\varphi_{23}\cos\lambda_3\sin\lambda_2 - \sin\lambda_3\cos\lambda_2\right) + \rho_3\dot{\phi}_3\sin\varphi_{23}\sin\lambda_2\cos\lambda_3 - \rho_3\dot{\eta}_3\left(\cos\varphi_{23}\sin\lambda_2\sin\lambda_3 + \cos\lambda_2\cos\lambda_3\right)}{c^4\tau_{23}\rho_2}\\
&- \frac{e_2^2}{m_2}\frac{\ddot{\eta}_2}{c^3} \equiv G_{\eta_2};
\end{aligned}$$

$$\dot{r}_3 = \frac{e_3 e_1}{m_3}\frac{1}{c{\rho_3}^2} - \frac{e_2 e_3}{m_3}\frac{\rho_3 - \rho_2\left(\cos\varphi_{32}\cos\lambda_2\cos\lambda_3 + \sin\lambda_2\sin\lambda_3\right)}{c^3}\frac{\left\langle \vec{\xi}^{(32)}, \dot{\vec{u}}^{(2)}\right\rangle}{c^3{\tau_{32}}^3} \quad (3)$$
$$- \frac{e_3 e_2}{m_3}\frac{\dot{r}_2\left(\cos\varphi_{32}\cos\lambda_2\cos\lambda_3 + \sin\lambda_2\sin\lambda_3\right) + \rho_2\dot{\phi}_2\sin\varphi_{32}\cos\lambda_2\cos\lambda_3 + \rho_2\dot{\eta}_2\left(\sin\lambda_3\cos\lambda_2 - \cos\varphi_{32}\sin\lambda_2\cos\lambda_3\right)}{c^4\tau_{23}} \quad (4)$$
$$- \frac{e_3^2}{m_3}\frac{\ddot{r}_3}{c^3} \equiv G_{r_3}; \quad (5)$$

(6)

$$\dot{\phi}_3 = \frac{e_3 e_2}{m_3}\left[\frac{\rho_2 \sin\varphi_{32}\cos\lambda_2}{{\tau_{32}}^3\rho_3\cos\lambda_3}\frac{\left\langle \vec{\xi}^{(32)}, \dot{\vec{u}}^{(2)}\right\rangle}{c^6} + \frac{\dot{r}_2\cos\lambda_2\sin\varphi_{32} + \rho_2\dot{\phi}_2\cos 2\cos\varphi_{32} - \rho_2\dot{\eta}_2\sin\lambda_2\sin\varphi_{32}}{c^4\tau_{32}\rho_3\cos\lambda_3} - \frac{\ddot{\phi}_3}{c^3}\right] = G_{\phi_3}$$

$$\begin{aligned}\dot{\eta}_3 =& \frac{e_3 e_2}{m_3}\frac{\rho_2\cos\varphi_{32}\sin\lambda_3\cos\lambda_2}{c^3\tau_{32}^3}\frac{\left\langle \vec{\xi}(32), \dot{\vec{u}}^{(2)}\right\rangle}{c^3\rho_3}\\ &+ \frac{e_3 e_2}{m_3 c^3}\frac{\dot{r}_3\left(\cos\varphi_{32}\cos\lambda_2\sin\lambda_3 - \sin\lambda_2\cos\lambda_3\right) + \rho_2\dot{\phi}_2\sin\varphi_{32}\sin\lambda_3\cos\lambda_2 - \rho_2\dot{\eta}_2\left(\cos\varphi_{23}\sin\lambda_2\sin\lambda_3 + \cos\lambda_2\cos\lambda_3\right.}{c\tau_{32}\rho_3}\\ &- \frac{e_3^2}{m_3}\frac{\ddot{\eta}_3}{c^3} \equiv G_{\eta_3}\end{aligned}$$

where

$$\dot{\rho}_n = r_n, \dot{\varphi}_n = \phi_n, \dot{\lambda}_n = \eta_n (n = 2,3) \Rightarrow \rho_n(t) = \rho_{n0} + \int_0^t r_n(s)ds,$$
$$\varphi_n = \varphi_{n0} + \int_0^t \phi_n(s)ds, \lambda_n = \lambda_{n0} + \int_0^t \eta_n(s)ds.$$

In [4] we have proved the existence-uniqueness of T-periodic solution of (3).

We consider these functions $\dot{\rho}_n = r_n, \dot{\varphi}_n = \phi_n, \dot{\lambda}_n = \eta_n$ as known ones. Then we must solve the spin equations (2) using spherical coordinates.

Recall some denotations from [3]:

$$\vec{\theta}^{(k)} = \left(\theta_1^{(k)}(t), \theta_2^{(k)}(t), \theta_3^{(k)}(t)\right), \vec{\sigma}^{(k)} = \left(\sigma_1^{(k)}(t), \sigma_2^{(k)}(t), \sigma_3^{(k)}(t)\right),$$
$$\vec{\theta}^{(k)} = \frac{1}{c}\left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}\right) = \frac{1}{c}\begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \frac{1}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix}\vec{e}_1 - \frac{1}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix}\vec{e}_2 + \frac{1}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix}\vec{e}_3$$

and then the spin tensors $\sigma_{\mu\nu}^{(k)}, (k = 1,2,3)$ are

$$\sigma_{\mu\nu}^{(k)} = \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & i\theta_1^{(k)} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & i\theta_2^{(k)} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & i\theta_3^{(k)} \\ -i\theta_1^{(k)} & -i\theta_2^{(k)} & -i\theta_3^{(k)} & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & \frac{i}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & -\frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & \frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \\ -\frac{i}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} & \frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} & -\frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} & 0 \end{pmatrix}.$$

We have proved in [4] that the last three equations in (2) are consequence of the first three ones in a weak sense so that we consider just the first three equations for every particle $(k = 1,2,3)$:

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1,n\neq k}^{3} \left(F_{1m}^{(kn)} \sigma_{m2}^{(k)} - F_{m2}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m2}^{(k)} - F_{m2}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{7}$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1,n\neq k}^{3} \left(F_{1m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{8}$$

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1,n\neq k}^{3} \left(F_{2m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{2m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{2m}^{(k)} \right). \tag{9}$$

We transform the last equations in the form:

$$\frac{d\sigma_\alpha{}^{(k)}}{ds_k} = F_{\alpha,L}^{(k)} + F_{\alpha,rad}^{(k)}, (\alpha = 1,2,3), (k = 1,2,3) \tag{10}$$

which consists of 9 equations for 9 unknown spin functions

$$\left(\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}, \sigma_1^{(3)}, \sigma_2^{(3)}, \sigma_3^{(3)} \right)$$

where

$$\begin{aligned} F_{\alpha,L}^{(k)} = \sum_{n=1,n\neq k}^{3} \frac{e_k e_n}{m_k c^2} \Big[& \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \vec{P}^{(kn)} - \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \vec{\xi}^{(kn)} \\ & + \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \right) \vec{\lambda}^{(k)} - \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle \right) \vec{\sigma}^{(k)} \Big]; \end{aligned}$$

$$\begin{aligned} F_{\alpha,rad}^{(k)} &= \frac{e_k{}^2}{m_k c^2} \left(-\frac{\Delta_k \left\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)} \right\rangle}{\Delta_k{}^4} \ddot{u}_\alpha^{(k)} + \frac{\Delta_k \left\langle \vec{\sigma}^{(k)}, \dddot{\vec{u}}^{(k)} \right\rangle - \left\langle \vec{\sigma}^{(k)}, \dddot{\vec{u}}^{(k)} \right\rangle}{\Delta_k{}^4} u_\alpha{}^{(k)} + \frac{\left\langle \vec{u}^{(k)}, \dddot{\vec{u}}^{(k)} \right\rangle}{\Delta_k{}^4} \sigma_\alpha{}^{(k)} \right) \\ &\approx \frac{e_k{}^2}{m_k c^2} \left(-\frac{\left\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)} \right\rangle}{c^3} \ddot{u}_\alpha^{(k)} + \frac{\left\langle \vec{\sigma}^{(k)}, \dddot{\vec{u}}^{(k)} \right\rangle}{c^3} u_\alpha{}^{(k)} + \frac{\left\langle \vec{u}^{(k)}, \dddot{\vec{u}}^{(k)} \right\rangle}{c^4} \sigma_\alpha{}^{(k)} \right) \end{aligned}$$

and

$$\begin{aligned} M_{kn} &= 1 + \left\langle \xi^{(kn)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 \approx \frac{c^2 + \left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle - \tau_{kn} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{c^2}; \\ L_{kn} &= -\frac{M_{kn} \Delta_{kn}{}^2}{c^6 \tau_{kn}{}^3} - \frac{D_{kn} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}{}^2 c^4 \tau_{kn}{}^2} \approx \frac{-c^2 - \left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle}{c^6 \tau_{kn}{}^3} \\ P_\alpha{}^{(kn)} &= \frac{M_{kn} \Delta_{kn}{}^2 u_\alpha^{(n)}}{\left(c^2 \tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} + \frac{\Delta_{kn}{}^2}{\left(c^2 \tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2} \left(\dot{u}_\alpha^{(n)} + \frac{u_\alpha^{(n)} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}{}^2} \right) \\ &\approx \frac{M_{kn} u_\alpha^{(n)}}{c^4 \tau_{kn}{}^3} + \frac{c^2 \dot{u}_\alpha^{(n)} + u_\alpha^{(n)} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{c^4 \tau_{kn}{}^2}. \end{aligned}$$

The coefficient before unknown functions are functions of velocities and trajectories of the moving particles. We have proved however an existence-uniqueness of a periodic solution of (3). Therefore, we can substitute these solutions into coefficient of (7,8,9) or (10). So, we obtain a system with known coefficient. It remains to prove an existence of periodic solution of (10).

3. Equations of Motion in Spherical Coordinates

To formulate th3D-Kepler form of the equations we need spherical coordinates:

$$x_1^{(n)}(t) = \rho_n(t)\cos\varphi_n(t)\cos\lambda_n(t); x_2^{(n)}(t) = \rho_n(t)\sin\varphi_n(t)\cos\lambda_n(t); x_3^{(n)}(t) = \rho_n(t)\sin\lambda_n(t)$$
$$\rho_n \geq 0; \varphi_n \geq 0; \lambda_n \in \left[-\frac{\pi}{2}+\delta, \frac{\pi}{2}-\delta\right], 0<\delta<\frac{\pi}{2}; u_\alpha^{(1)} = 0, (\alpha = 1,2,3); \tau_{kn} = \frac{1}{c}\rho_n\left(t-\tau_{kn}\right);$$
$$u_1^{(n)} = \dot{\rho}_n\cos\varphi_n\cos\lambda_n - \rho_n\dot{\varphi}_n\sin\varphi_n\cos\lambda_n - \rho_n\dot{\lambda}_n\cos\varphi_n\sin\lambda_n;$$
$$u_2^{(n)} = \dot{\rho}_n\sin\varphi_n\cos\lambda_n + \rho_n\dot{\varphi}_n\cos\varphi_n\cos\lambda_n - \rho_n\dot{\lambda}_n\sin\varphi_n\sin\lambda_n;$$
$$u_3^{(n)} = \dot{\rho}_n\sin\lambda_n + \rho_n\dot{\lambda}_n\cos\lambda_n;$$
$$\dot{u}_1^{(n)} \approx \ddot{\rho}_n\cos\varphi_n\cos\lambda_n - \rho_n\ddot{\varphi}_n\sin\varphi_n\cos\lambda_n - \rho_n\ddot{\lambda}_n\cos\varphi_n\sin\lambda_n;$$
$$\dot{u}_2^{(n)} \approx \ddot{\rho}_n\sin\varphi_n\cos\lambda_n + \rho_n\ddot{\varphi}_n\cos\varphi_n\cos\lambda_n - \rho_n\ddot{\lambda}_n\sin\varphi_n\sin\lambda_n;$$
$$\dot{u}_3^{(n)} \approx \ddot{\rho}_n\sin\lambda_n + \rho_n\ddot{\lambda}_n\cos\lambda_n;$$
$$\ddot{u}_1^{(n)} \approx \dddot{\rho}_n\cos\varphi_n\cos\lambda_n - \dddot{\varphi}_n\rho_n\sin\varphi_n\cos\lambda_n - \dddot{\lambda}_n\rho_n\cos\varphi_n\sin\lambda_n;$$
$$\ddot{u}_2^{(n)} \approx \dddot{\rho}_n\sin\varphi_n\cos\lambda_n + \dddot{\varphi}_n\rho_n\cos\varphi_n\cos\lambda_n - \dddot{\lambda}_n\rho_n\sin\varphi_n\sin\lambda_n;$$
$$\ddot{u}_3^{(n)} \approx \dddot{\rho}_n\sin\lambda_n + \dddot{\lambda}_n\rho_n\cos\lambda_n.$$

Let us write the system of spin equations in detail:

$$\begin{aligned}
\frac{d\sigma_1^{(1)}(t)}{dt} &= \sum_{n=1,n\neq 1}^{3}\frac{e_1e_n}{m_1c^2}\left(\left\langle\vec{\xi}^{(1n)},\vec{\sigma}^{(1)}\right\rangle P_1^{(1n)} - \left\langle\vec{P}^{(1n)},\vec{\sigma}^{(2)}\right\rangle\xi_1^{(1n)}\right) \equiv H_1^{(1)},\\
\frac{d\sigma_2{}^{(1)}(t)}{dt} &= \sum_{n=1,n\neq 1}^{3}\frac{e_1e_n}{m_1c^2}\left(\left\langle\vec{\xi}^{(1n)},\vec{\sigma}^{(1)}\right\rangle P_2^{(1n)} - \left\langle\vec{P}^{(1n)},\vec{\sigma}^{(2)}\right\rangle\xi_2^{(1n)}\right) \equiv H_2^{(1)},\\
\frac{d\sigma_3{}^{(1)}(t)}{dt} &= \sum_{n=1,n\neq 1}^{3}\frac{e_1e_n}{m_1c^2}\left(\left\langle\xi^{(1n)},\vec{\sigma}^{(1)}\right\rangle P_3^{(1n)} - \left\langle\vec{P}^{(1n)},\vec{\sigma}^{(2)}\right\rangle\xi_3^{(1n)}\right) \equiv H_3^{(1)},
\end{aligned} \tag{11}$$

$$\begin{aligned}
\frac{d\sigma_1^{(2)}(t)}{dt} = &\sum_{n=1,n\neq 2}^{3}\frac{e_2e_n}{m_2c^2}\Bigg[\left\langle\vec{\xi}^{(2n)},\vec{\sigma}^{(2)}\right\rangle P_1^{(2n)} - \left\langle\vec{P}^{(2n)},\vec{\sigma}^{(2)}\right\rangle\xi_1^{(2n)}\\
&+\left(\tau_{2n}\left\langle\vec{P}^{(2n)},\vec{\sigma}^{(2)}\right\rangle + L_{2n}\left\langle\vec{\xi}^{(2n)},\vec{\sigma}^{(2)}\right\rangle\right)\frac{u_1^{(2)}}{c} - \left(\tau_{2n}\left\langle\vec{P}^{(2n)},\vec{\lambda}^{(2)}\right\rangle + L_{2n}\left\langle\vec{\xi}^{(2n)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right)\sigma_1^{(2)}\Bigg]\\
&+\frac{e_2^2}{m_2c^2}\left(-\frac{\left\langle\vec{u}^{(2)},\vec{\sigma}^{(2)}\right\rangle}{c^3}\ddot{u}_1^{(2)} + \frac{\left\langle\vec{\sigma}^{(2)},\dddot{\vec{u}}^{(2)}\right\rangle}{c^3}u_1^{(2)} + \frac{\left\langle\vec{u}^{(2)},\ddot{u}^{(2)}\right\rangle}{c^4}\sigma_1^{(2)}\right) \equiv H_1^{(2)},
\end{aligned}$$

$$\begin{aligned}
\frac{d\sigma_2^{(2)}(t)}{dt} = &\sum_{n=1,n\neq 2}^{3}\frac{e_2e_n}{m_2c^2}\Bigg[\left\langle\vec{\xi}^{(2n)},\vec{\sigma}^{(2)}\right\rangle P_2^{(2n)} - \left\langle\vec{P}^{(2n)},\vec{\sigma}^{(2)}\right\rangle\xi_2^{(2n)}\\
&+\left(\tau_{2n}\left\langle\vec{P}^{(2n)},\vec{\sigma}^{(2)}\right\rangle + L_{2n}\left\langle\vec{\xi}^{(2n)},\vec{\sigma}^{(2)}\right\rangle\right)\frac{u_2^{(2)}}{c} - \left(\tau_{2n}\left\langle\vec{P}^{(2n)},\vec{\lambda}^{(2)}\right\rangle + L_{2n}\left\langle\vec{\xi}^{(2n)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right)\sigma_2^{(2)}\Bigg]\\
&+\frac{e_2^2}{m_2c^2}\left(-\frac{\left\langle\vec{u}^{(2)},\vec{\sigma}^{(2)}\right\rangle}{c^3}\ddot{u}_2^{(2)} + \frac{\left\langle\vec{\sigma}^{(2)},\dddot{\vec{u}}^{(2)}\right\rangle}{c^3}u_2^{(2)} + \frac{\left\langle\vec{u}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle}{c^4}\sigma_2^{(2)}\right) \equiv H_2^{(2)},
\end{aligned}$$

$$\frac{d\sigma_3^{(2)}(t)}{dt} = \sum_{n=1,n\neq 2}^{3} \frac{e_2 e_n}{m_2 c^2} \left[\left\langle \vec{\xi}^{(2n)}, \vec{\sigma}^{(2)} \right\rangle P_3^{(2n)} - \left\langle \vec{P}^{(2n)}, \vec{\sigma}^{(2)} \right\rangle \xi_3^{(2n)} \right.$$
$$+ \left(\tau_{2n} \left\langle \vec{P}^{(2n)}, \vec{\sigma}^{(2)} \right\rangle + L_{2n} \left\langle \vec{\xi}^{(2n)}, \vec{\sigma}^{(2)} \right\rangle \right) \frac{u_3^{(2)}}{\Delta_2} - \left. \left(\tau_{2n} \left\langle \vec{P}^{(2n)}, \vec{\lambda}^{(2)} \right\rangle + L_{2n} \left\langle \vec{\xi}^{(2n)}, \frac{\vec{u}^{(2)}}{\Delta_2} \right\rangle \right) \sigma_3^{(2)} \right]$$
$$+ \frac{e_2^2}{m_2 c^2} \left(- \frac{\left\langle \vec{u}^{(2)}, \vec{\sigma}^{(2)} \right\rangle}{c^3} \ddot{u}_3^{(2)} + \frac{\left\langle \vec{\sigma}^{(2)}, \ddot{\vec{u}}^{(2)} \right\rangle}{c^3} u_3^{(2)} + \frac{\left\langle \vec{u}^{(2)}, \ddot{\vec{u}}^{(2)} \right\rangle}{c^4} \sigma_3^{(2)} \right) \equiv H_3^{(2)},$$

$$\frac{d\sigma_1^{(3)}(t)}{dt} = \sum_{n=1,n\neq 3}^{3} \frac{e_2 e_n}{m_2 c^2} \left[\left\langle \vec{\xi}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle P_1^{(3n)} - \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \xi_1^{(3n)} \right.$$
$$+ \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle + L_{3n} \left\langle \vec{\xi}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \right) \frac{u_1^{(3)}}{c} - \left. \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\lambda}^{(3)} \right\rangle + L_{3n} \left\langle \vec{\xi}^{(3n)}, \frac{\vec{u}^{(3)}}{c} \right\rangle \right) \sigma_1^{(3)} \right]$$
$$+ \frac{e_3^2}{m_3 c^2} \left(- \frac{\left\langle u^{(3)}, \vec{\sigma}^{(3)} \right\rangle}{c^3} \ddot{u}_1^{(3)} + \frac{\left\langle \bar{\sigma}^{(3)}, \ddot{u}^{(3)} \right\rangle}{c^3} u_1^{(3)} + \frac{\left\langle u^{(3)}, \ddot{u}^{(3)} \right\rangle}{c^4} \sigma_1^{(3)} \right) \equiv H_1^{(3)},$$

$$\frac{d\sigma_2^{(3)}(t)}{dt} = \sum_{n=1,n=3}^{3} \frac{e_2 e_n}{m_2 c^2} \left[\left\langle \vec{\xi}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle P_2^{(3n)} - \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \xi_2^{(3n)} \right.$$
$$+ \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle + L_{3n} \left\langle \xi^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \right) \frac{u_2^{(3)}}{c} - \left. \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\lambda}^{(3)} \right\rangle + L_{3n} \left\langle \vec{\xi}^{(3n)}, \frac{\vec{u}^{(3)}}{c} \right\rangle \right) \sigma_2^{(3)} \right]$$
$$+ \frac{e_3^2}{m_3 c^2} \left(- \frac{\left\langle u^{(3)}, \vec{\sigma}^{(3)} \right\rangle}{c^3} \ddot{i}_2^{(3)} + \frac{\left\langle \vec{\sigma}^{(3)}, \ddot{u}^{(3)} \right\rangle}{c^3} u_2^{(3)} + \frac{\left\langle \vec{u}^{(3)}, \ddot{u}^{(3)} \right\rangle}{c^4} \sigma_2^{(3)} \right) \equiv H_2^{(3)},$$

$$\frac{d\sigma_3^{(3)}(t)}{dt} = \sum_{n=1,n\neq 3}^{3} \frac{e_2 e_n}{m_2 c^2} \left[\left\langle \vec{\xi}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle P_3^{(3n)} - \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \xi_3^{(3n)} \right.$$
$$+ \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle + L_{3n} \left\langle \vec{\xi}^{(3n)}, \vec{\sigma}^{(3)} \right\rangle \right) \frac{u_3^{(3)}}{c} - \left. \left(\tau_{3n} \left\langle \vec{P}^{(3n)}, \vec{\lambda}^{(3)} \right\rangle + L_{3n} \left\langle \vec{\xi}^{(3n)}, \frac{\vec{u}^{(3)}}{c} \right\rangle \right) \sigma_3^{(3)} \right]$$
$$+ \frac{e_3^2}{m_3 c^2} \left(- \frac{\left\langle \tilde{u}^{(3)}, \vec{\sigma}^{(3)} \right\rangle}{c^3} \ddot{u}_3^{(3)} + \frac{\left\langle \vec{\sigma}^{(3)}, \ddot{u}^{(3)} \right\rangle}{c^3} u_3^{(3)} + \frac{\left\langle \bar{u}^{(3)}, \ddot{\vec{u}}^{(3)} \right\rangle}{c^4} \sigma_3^{(3)} \right) = H_3^{(3)}.$$

4. Estimates of the Right-Hand Sides of the Spin Equations

We estimate the right-hand sides of the above system (11) considering the presentation of the above expressions in spherical coordinates and the inequalities

$$\begin{aligned}\vec{\xi}^{(kn)} &= (\rho_k(t) \cos\varphi_k(t) \cos\lambda_k(t) - \rho_n(t-\tau_{kn}) \cos\varphi_n(t-\tau_{kn})) \cos\lambda_n(t-\tau_{kn}) \Big),\\ &\rho_k(t) \sin\varphi_k(t) \cos\lambda_k(t) - \rho_n(t-\tau_{kn})) \sin\varphi_n(t-\tau_{kn})) \cos\lambda_n(t-\tau_{kn})),\\ &\rho_k(t) \sin\lambda_k(t) - \rho_n(t-\tau_{kn})) \sin\lambda_n(t-\tau_{kn}))).\end{aligned}$$

Let us note that the functions with retarded arguments are substituted by the initial functions (cf. [4]) and $\rho_1(t) \equiv 0$. We need the formulas

$$\left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle = \dot{\rho}_n \ddot{\rho}_n + \rho_n^2 \dot{\varphi}_n \ddot{\varphi}_n \cos\lambda_n + \rho_n^2 \dot{\lambda}_n \ddot{\lambda}_n, \left\langle \vec{u}^{(n)}, \ddot{\vec{u}}^{(n)} \right\rangle = \dddot{\rho}_n \dot{\rho}_n + \dddot{\varphi}_n \rho_n^2 \dot{\varphi}_n \cos^2\lambda_n + \dddot{\lambda}_n \rho_n^2 \dot{\lambda}_n,$$

$$\left\langle \vec{u}^{(n)}, \vec{u}^{(n)} \right\rangle = r_n^2 + \rho_n^2 \phi_n^2 \cos^2\lambda_n + \rho_n^2 \eta_n^2 \le \left(R_n^2 + \rho_n^2 \Phi_n^2 + \rho_n^2 Y_n^2 \right) e^{2\mu\pi} \le \bar{c}^2 < c^2, (n = 2, 3),$$

$$\left| \left\langle \vec{u}^{(2)}, \dot{\vec{u}}^{(2)} \right\rangle \right| \le \omega R_2^2 e^{2\mu T} + \omega \rho_2^2 {\Phi_2}^2 e^{2\mu T} + \omega \rho_2^2 Y_2^2 e^{2\mu T} \le \omega \bar{c}^2,$$

$$\left\| \dot{\vec{u}}^{(n)} \right\| = \sqrt{\left\langle \dot{\vec{u}}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle} = \sqrt{\ddot{\rho}_n{}^2 + \rho_n^2 \ddot{\varphi}_n{}^2 \cos^2\lambda_n + {\rho_n}^2 \ddot{\lambda}_n^2} \le e^{\mu T} \bar{c}\omega,$$

$$c^2 \dot{u}_\alpha^{(2)} + \left\langle \vec{u}^{(2)}, \dot{\vec{u}}^{(2)} \right\rangle u_\alpha^{(2)} \le 3c^2 \bar{c} + \omega \left(\bar{c} R_2^2 e^{2\mu T} + \rho_2^2 \Phi_2^2 e^{2\mu T} + \rho_2^2 Y_2^2 e^{2\mu T} \right) \le 3c^2 \bar{c} + 3\omega \bar{c}^2,$$

$$\left\| \ddot{\vec{u}}^{(n)} \right\| = \sqrt{\left\langle \ddot{\vec{u}}^{(n)}, \ddot{\vec{u}}^{(n)} \right\rangle} = \sqrt{\dddot{\rho}_n{}^2 + \dddot{\varphi}_n{}^2 {\rho_n}^2 \cos\lambda_n + \dddot{\lambda}_n^2 {\rho_n}^2} \le e^{\mu T} \omega^2 \sqrt{{R_n}^2 + {\rho_n}^2 {\Phi_n}^2 + {\rho_n}^2 {Y_n}^2} \le e^{\mu T} \omega^2 \bar{c},$$

$$|M_{kn}| \le \left| 1 + \left\langle \xi^{(k)_0}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 \right| = \left| \frac{c^2 + \left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle - \tau_{\text{fn}} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{c^2} \right| \le 1 + 2\tau_{k_2} \beta e^{\mu T} \bar{c}\omega,$$

$$|L_{kn}| \le \frac{c + \tau_{kn} \left\| \dot{\vec{u}}^{(n)} \right\|}{c^5 {\tau_{kn}}^3} = \frac{1}{c^4 {\tau_{kn}}^3} + \frac{\left\| \dot{\vec{u}}^{(n)} \right\|}{c^5 {\tau_{kn}}^2} \le \frac{1}{c^4 {\tau_{kn}}^3} + \frac{e^{\mu T} \beta\omega}{c^4 {\tau_{kn}}^2}.$$

Recall the inequalities from [4]:

$$\begin{aligned} \rho_{kn}(t) &= \sqrt{(\rho_k \cos\varphi_k \cos\lambda_k - \rho_n \cos\varphi_n \cos\lambda_n)^2 + (\rho_k \sin\varphi_k \cos\lambda_k - \rho_n \sin\varphi_n \cos\lambda_n)^2 + (\rho_k \sin\lambda_k - \rho_n \sin\lambda_n)^2} \\ &\ge \sqrt{{\rho_k}^2(t) + {\rho_n}^2 (t - \tau_{kn}) - 2\rho_k(t) \rho_n (t - \tau_{kn})} \\ &= |\rho_k(t) - \rho_n (t - \tau_{kn})| \ge |\rho_{k0} - \rho_{n0}| - (R_k + R_n) \frac{e^{\mu T} - 1}{\mu} = \Delta_{kn} > 0; \end{aligned}$$

$$\left| P_\alpha^{(kn)} \right| \le \frac{1 + 2\tau_{kn} \beta e^{\mu T} \bar{c}\omega}{c^3 {\tau_{kn}}^3} + \frac{\left(c^2 + \bar{c}^2 \right) \left\| \dot{\vec{u}}^{(n)} \right\|}{c^4 {\tau_{kn}}^2} \le \frac{1}{c^3 {\tau_{kn}}^3} + \frac{\left(2\beta^2 + \bar{c} \right) e^{\mu T} \omega}{c^2 {\tau_{kn}}^2} \approx \frac{1}{c^3 {\tau_{kn}}^3} + \frac{\bar{c} e^{\mu T} \omega}{c^2 {\tau_{kn}}^2};$$

$$\begin{aligned} \tau_{kn}(t) &= \frac{1}{c} \sqrt{\sum_{\alpha=1}^3 \left[x_\alpha^{(k)}(t) - x_\alpha^{(n)} \left(t - \tau_{kn}(t) \right] \right.^2} \\ &= \frac{1}{c} \sqrt{\rho_k^2 + \rho_n^2 - 2\rho_k \rho_n \left(\cos\lambda_k \cos\lambda_n \cos(\varphi_k - \varphi_n) + \sin\lambda_k \sin\lambda_n \right)} \\ &\ge \frac{1}{c} \sqrt{{\rho_k}^2 + \rho_n^2 - 2\rho_k \rho_n \left| \cos\lambda_k \cos\lambda_n \cos(\varphi_k - \varphi_n) + \sin\lambda_k \sin\lambda_n \right|} \\ &\ge \frac{1}{c} \sqrt{\rho_k^2 + \rho_n^2 - 2\rho_k \rho_n} = \frac{1}{c} \sqrt{(\rho_k - \rho_n)^2} = \frac{|\rho_k(t) - \rho_n (t - \tau_{kn})|}{c} \\ &\Leftrightarrow \frac{1}{\tau_{kn}} \le \frac{c}{|\rho_k(t) - \rho_n (t - \tau_{kn})|} \ge \frac{c}{|\rho_{k0} - \rho_{n0}| - (R_k + R_n) \left(e^{\mu T} - 1 \right) / \mu} \equiv \frac{c}{\Delta_{kn}}. \end{aligned}$$

For the right-hand sides we obtain:

$$\left|H_1^{(1)}\left(t,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)\right| \le \frac{|e_1e_2|}{m_1c^2}\left(\left|\left\langle \vec{\xi}^{(12)},\vec{\sigma}^{(1)}\right\rangle P_1^{(12)}\right| + \left|\left\langle \vec{P}^{(12)},\vec{\sigma}^{(2)}\right\rangle \xi_1^{(1)}\right|\right)$$
$$+\frac{|e_1e_3|}{m_1c^2}\left(\left|\left\langle \vec{\xi}^{(13)},\vec{\sigma}^{(1)}\right\rangle P_1^{(13)}\right| + \left|\left\langle \vec{P}^{(13)},\vec{\sigma}^{(2)}\right\rangle \xi_1^{(1)}\right|\right)$$
$$\le \frac{|e_1e_2|}{m_1c^2}4c\tau_{12}\left(\frac{1}{c^3{\tau_{12}}^3}+\frac{\beta e^{\mu T}\omega}{c{\tau_{12}}^2}\right)\sqrt{\sum_{\gamma=1}^3\left(\sigma_\gamma^{(1)}\right)^2} + \frac{|e_1e_3|}{m_1c},\left[4c\tau_{13}\left(\frac{1}{c^3{\tau_{13}}^3}+\frac{\beta e^{\mu T}\omega}{c{\tau_{13}}^2}\right)\right]\sqrt{\sum_{y=1}^3\left(\sigma_\gamma^{(1)}\right)^2}$$
$$\le \frac{4}{m_1c^2}\left[|e_1e_2|\left(\frac{1}{c^2{\tau_{12}}^2}+\frac{\beta e^{\mu T}\omega}{\tau_{12}}\right)+|e_1e_3|\left(\frac{1}{c^2{\tau_{13}}^2}+\frac{\beta e^{\mu T}\omega}{c\tau_{13}}\right)\right]\sqrt{\sum_{\gamma=1}^3\left(\sigma_\gamma^{(1)}\right)^2}$$
$$\le \frac{4\,|e_1e_2|}{m_1c^2}\left[\frac{1}{{\Delta_{12}}^2}+\frac{\bar{e}e^{\mu T}\omega}{\Delta_{12}}+\frac{1}{{\Delta_{13}}^2}+\frac{\bar{e}e^{\mu T}\omega}{\Delta_{13}}\right]\sqrt{\sum_{\gamma=1}^3\left(\sigma_\gamma^{(1)}\right)^2}.$$

Since the first particle is stated at the origin $\rho_1 = 0$, then

$$\left|H_2^{(1)}\left(t,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)\right| \le \frac{4\,|e_1e_2|}{m_1c^2}\left(\frac{1}{{\Delta_{21}}^2}+\frac{\bar{c}e^{\mu T}\omega}{\Delta_{21}}+\frac{1}{{\Delta_{23}}^2}+\frac{\bar{c}e^{\mu T}\omega}{\Delta_{23}}\right)\sqrt{\sum_{\gamma=1}^3\left(\sigma_\gamma^{(1)}\right)^2},$$
$$\left|H_3^{(1)}\left(t,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)\right| \le \frac{4\,|e_1e_2|}{m_1c^2}\left(\frac{1}{{\Delta_{31}}^2}+\frac{\bar{c}e^{\mu T}\omega}{\Delta_{31}}+\frac{1}{{\Delta_{32}}^2}+\frac{\bar{c}e^{\mu T}\omega}{\Delta_{32}}\right)\sqrt{\sum_{\gamma=1}^3\left(\sigma_\gamma^{(1)}\right)^2}.$$

For the next components we have:

$$\left|H_1^{(2)}\right| \le \frac{|e_2e_1|}{m_2c^2}\left[\left|\left\langle \vec{\xi}^{(21)},\vec{\sigma}^{(2)}\right\rangle P_1^{(21)}\right| + \left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle \xi_1^{(2)}\right|\right.$$
$$+\left(\tau_{21}\left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle\right| + |L_{21}|\left|\left\langle \vec{\xi}^{(21)},\vec{\sigma}^{(2)}\right\rangle\right|\right)\frac{\left|u_1^{(2)}\right|}{c} + \left.\left(\tau_{21}\left|\left\langle \vec{P}^{(21)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right| + |L_{21}|\left|\left\langle \vec{\xi}^{(21)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right||\sigma_1^{(2)}|\right)\right]$$
$$+\frac{|e_2e_3|}{m_2c^2}\left[\left|\left\langle \vec{\xi}^{(23)},\vec{\sigma}^{(2)}\right\rangle P_1^{(2)}\right| + \left|\left\langle \vec{P}^{(23)},\vec{\sigma}^{(2)}\right\rangle \xi_1^{(23)}\right| + \left(\tau_{23}\left|\left\langle \vec{P}^{(2)},\vec{\sigma}^{(2)}\right\rangle\right| + |L_{23}|\left\langle \vec{\xi}^{(23)},\vec{\sigma}^{(2)}\right\rangle\right),\frac{\left|u_1^{(2)}\right|}{c}\right.$$
$$+\left.\left(\tau_{23}\left|\left\langle \vec{P}^{(23)},\frac{\vec{u}^{(2)}}{c}\right\rangle + \right| L_{23}||\left\langle \vec{\xi}^{(2n)},\frac{\vec{u}^{(2)}}{c}\right\rangle||\sigma_1^{(2)}\right)\right]$$
$$+\frac{e_2^2}{m_2c^2}\left(\frac{|\langle\vec{u}^{(2)},\vec{\sigma}^{(2)}\rangle|}{c^3}\left|\ddot{u}_1^{(2)}\right| + \frac{\left|\left\langle\vec{\sigma}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle\right|}{c^3}\left|u_1^{(2)}\right| + \frac{\left|\left\langle\vec{u}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle\right|}{c^4}\left|\sigma_1^{(2)}\right|\right)$$
$$\le \frac{|e_2e_1|}{m_2c^2}\left[4c\tau_{21}\left(\frac{1}{c^3{\tau_{21}}^3}+\frac{\beta e^{\mu\pi}\omega}{c{\tau_{21}}^2}\right)+\beta\left(3\tau_{21}\left(\frac{1}{c^3{\tau_{21}}^3}+\frac{\beta e^{\mu\pi}\omega}{c{\tau_{21}}^2}\right)+c\tau_{21}\left(\frac{1}{c^4{\tau_{21}}^3}+\frac{e^{\mu\tau}\beta\omega}{c^4{\tau_{21}}^2}\right)\right)\right]\sqrt{\sum_{\gamma=1}^3({\sigma_\gamma}^{22})^2}$$
$$+\frac{|e_2e_3|}{m_2c^2}\left[4c\tau_{23}\left(\frac{1}{c^3{\tau_{23}}^3}+\frac{\beta e^{\mu\tau}\omega}{c{\tau_{23}}^2}\right)+\beta\left(3\tau_{21}\left(\frac{1}{c^3{\tau_{23}}^3}+\frac{\beta e^{\mu\tau}\omega}{c{\tau_{23}}^2}\right)+c\tau_{21}\left(\frac{1}{c^4{\tau_{23}}^3}+\frac{e^{\mu\tau}\beta\omega}{c^4{\tau_{23}}^2}\right)\right)\right]\sqrt{\sum_{\gamma-1}^3\left(\sigma_\gamma^{(2)}\right)^2}$$
$$+\frac{{e_2}^2\bar{c}\left\|\ddot{\vec{u}}^{(2)}\right\|}{m_2c^2}\left(\frac{1}{c^3}+\frac{1}{c^3}+\frac{1}{c^4}\right)\sqrt{\sum_{\gamma=1}^3(\sigma_\gamma(2))^2}$$
$$\le \left[\frac{|e_2e_1|}{m_2c^2}\left(\frac{4}{c^2{\tau_{21}}^2}+\frac{4\beta}{c^3{\tau_{21}}^2}+\frac{4\beta e^{\mu T}\omega}{\tau_{21}}+\frac{3e^{\mu\tau}\beta^2\omega}{c\tau_{21}}\right)+\frac{|e_2e_3|}{m_2c^2}\left(\frac{4}{c^2{\tau_{23}}^2}+\frac{4\beta}{c^3{\tau_{23}}^2}+\frac{4\beta e^{\mu\tau}\omega}{\tau_{23}}+\frac{3e^{\mu T}\beta^2\omega}{c\tau_{23}}\right)\right]\sqrt{\sum_{\gamma-1}^3\left(\sigma_\gamma^{(2)}\right)^2}$$
$$+\frac{e_2^2e^{\mu T}\omega^2\bar{c}^2}{m_2c^2}\left(\frac{1}{c^3}+\frac{1}{c^3}+\frac{1}{c^4}\right)\sqrt{\sum_{\gamma-1}^3\left(\sigma_\gamma^{(2)}\right)^2}$$

$$\approx \frac{|e_2e_1|}{m_2c^2}\left[\frac{4}{c^2{\tau_{21}}^2}+\frac{4\beta e^{\mu T}\omega}{\tau_{21}}+\frac{4}{c^2{\tau_{23}}^2}+\frac{4\beta e^{\mu T}\omega}{\tau_{23}}+\frac{2e^{\mu}T\omega^2\bar{c}^2}{c^3}\right]\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(2)}\right)^2}$$

$$\leq \frac{|e_2e_1|}{m_2c^2}\left(\frac{4}{\rho_2^2\left(t-\tau_{21}\right)}+\frac{4\bar{c}e^{\mu T}\omega}{\rho_2\left(t-\tau_{21}\right)}+\frac{4}{\left|\rho_2(t)-\rho_3\left(t-\tau_{23}\right)\right|^2}+\frac{4\bar{c}e^{\mu T}\omega}{\left|\rho_2(t)-\rho_3\left(t-\tau_{23}\right)\right|}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right)\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(2)}\right)^2}$$

$$\leq \frac{|e_2e_1|}{m_2c^2}\left(\frac{4}{{\Delta_{21}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{21}}+\frac{4}{{\Delta_{23}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{23}}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right)\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(2)}\right)^2};$$

$$\left|H_2^{(2)}\right| \leq \frac{|e_2e_1|}{m_2c^2}\left[\left|\left\langle \vec{\xi}^{(21)},\vec{\sigma}^{(2)}\right\rangle P_2^{(21)}\right|+\left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle \xi_2^{(21)}\right|\right.$$

$$+\left(\tau_{21}\left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle\right|+|L_{21}|\left|\left\langle \vec{\xi}^{(21)},\vec{\sigma}^{(2)}\right\rangle\right|\right),\frac{\left|u_2^{(2)}\right|}{c}+\left(\tau_{21}\left|\left\langle \vec{P}^{(21)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|+|L_{21}|\left|\left\langle \vec{\xi}^{(21)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|\right)\left|\sigma_2^{(2)}\right|\Bigg]$$

$$+\frac{|e_2e_3|}{m_2c^2}\left[\left|\left\langle \vec{\xi}^{(23)},\vec{\sigma}^{(2)}\right\rangle P_2^{(23)}\right|+\left|\left\langle \vec{P}^{(23)},\vec{\sigma}^{(2)}\right\rangle \xi_2^{(23)}\right|\right.$$

$$+\left(\tau_{23}\left|\left\langle \vec{P}^{(23)},\vec{\sigma}^{(2)}\right\rangle\right|+|L_{23}|\left|\left\langle \vec{\xi}^{(23)},\vec{\sigma}^{(2)}\right\rangle\right|\right),\frac{|u_2^{(2)}|}{c}+\left(\tau_{23}\left|\left\langle \vec{P}^{(23)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|+|L_{23}|\left|\left\langle \vec{\xi}^{(2n)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|\right)\left|\sigma_2^{(2)}\right|\Bigg]$$

$$+\frac{e_2^2}{m_2c^2}\left(\frac{|\langle \vec{u}^{(2)},\vec{\sigma}^{(2)}\rangle|}{c^3}\left|\ddot{u}_2^{(2)}\right|+\frac{\left|\left\langle \vec{\sigma}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle\right|}{c^3}\left|u_2^{(2)}\right|+\frac{\left|\left\langle \vec{u}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle\right|}{c^4}\left|\sigma_2^{(2)}\right|\right)$$

$$\leq \frac{|e_2e_1|}{m_2c^2}\left(\frac{4}{{\Delta_{21}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{21}}+\frac{4}{{\Delta_{23}}^2}+\frac{4\bar{c}\mu^{\mu T}\omega}{\Delta_{23}}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right)\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(2)}\right)^2};$$

$$\left|H_3^{(2)}\right| \leq \frac{|e_2e_1|}{m_2c^2}\left[\left|\left\langle \xi^{(21)},\vec{\sigma}^{(2)}\right\rangle P_3^{(21)}\right|+\left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle \xi_3^{(21)}\right|\right.$$

$$+\left(\tau_{21}\left|\left\langle \vec{P}^{(21)},\vec{\sigma}^{(2)}\right\rangle\right|+|L_{21}|\left|\left\langle \xi^{(21)},\vec{\sigma}^{(2)}\right\rangle\right|\right),\frac{\left|u_3^{(2)}\right|}{c}+\left(\tau_{21}\left|\left\langle \vec{P}^{(2)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|+|L_{21}|\left\langle\left\langle \vec{\xi}^{(2)},\frac{\vec{u}^{(2)}}{c}\right\rangle||\left|\sigma_3^{(2)}\right|\right]+$$

$$+\frac{|\xi_2e_3|}{m_2c^2}\left[\left|\left\langle \xi^{(23)},\vec{\sigma}^{(2)}\right\rangle P_3^{(23)}\right|+\left|\left\langle \vec{P}^{(23)},\vec{\sigma}^{(2)}\right\rangle \xi_3^{(23)}\right|\right.$$

$$+\left(\tau_{23}\left|\left\langle \vec{P}^{(23)},\vec{\sigma}^{(2)}\right\rangle\right|+|L_{23}|\left|\left\langle \vec{\xi}^{(23)},\vec{\sigma}^{(2)}\right\rangle\right|\right)\frac{|u_3^{(2)|}}{c}+\left(\tau_{23}\left|\left\langle \vec{P}^{(23)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|+|L_{23}|\left|\left\langle \vec{\xi}^{(2n)},\frac{\vec{u}^{(2)}}{c}\right\rangle\right|\right)|\sigma_3^{(2)}|\ ,\Bigg]$$

$$+\frac{e_2^2}{m_2c^2}\left(\frac{|\ \langle \vec{u}^{(2)},\vec{\sigma}^{(2)}\rangle}{c^3}\left|\dot{u}_3^{(2)}\right|+\frac{|\ \left\langle \vec{\sigma}^{(2)},\ddot{\vec{u}}^{(2)}\right\rangle}{c^3}\left|u_3^{(2)}\right|+\frac{|\ \langle \vec{u}^{(2)},\ddot{u}^{(2)}\rangle}{c^4}\left|\sigma_3^{(2)}\right|\right)$$

$$\leq \frac{|e_2e_1|}{m_2c^2}\left(\frac{4}{{\Delta_{21}}^2}+\frac{4\bar{c}e^{\mu\pi}\omega}{\Delta_{21}}+\frac{4}{{\Delta_{23}}^2}+\frac{4\bar{c}e^{\mu IT}\omega}{\Delta_{23}}+\frac{2e^{\mu\pi}\omega^2\bar{c}^2}{c^3}\right)\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(2)}\right)^2};$$

$$\left|H_1^{(3)}\right| \leq \frac{e_3e_1}{m_cc^2}\left[\left\langle \vec{\xi}^{(31)},\vec{\sigma}^{(3)}\right\rangle P_1^{(31)}-\left\langle \vec{P}^{(31)},\vec{\sigma}^{(3)}\right\rangle \xi_1^{(31)}+\left(\tau_{31}\left\langle \vec{P}^{(31)},\vec{\sigma}^{(3)}\right\rangle+L_{31}\left\langle \vec{\xi}^{(31)},\vec{\sigma}^{(3)}\right\rangle\right)\right.$$

$$\left.\frac{u_1^{(3)}}{c}-\left(\tau_{31}\left\langle \vec{P}^{(31)},\vec{\lambda}^{(3)}\right\rangle+L_{31}\left\langle \vec{\xi}^{(31)},\frac{\vec{u}^{(3)}}{c}\right\rangle\right)\sigma_1^{(3)}\right]+$$

$$+\frac{e_3e_2}{m_2c^2}\left[\left\langle \vec{\xi}^{(32)},\vec{\sigma}^{(3)}\right\rangle P_1^{(3)}-\left\langle \vec{P}^{(32)},\vec{\sigma}^{(3)}\right\rangle \xi_1^{(32)}+\left(\tau_{32}\left\langle \vec{P}^{(32)},\vec{\sigma}^{(3)}\right\rangle+L_{32}\left\langle \vec{\xi}^{(3n)},\vec{\sigma}^{(3)}\right\rangle\right)\frac{u_1^{(3)}}{c}\right.$$

$$\left.-\left(\tau_{32}\left\langle \vec{P}^{(32)},\vec{\lambda}^{(3)}\right\rangle+L_{32}\left\langle \vec{\xi}^{(32)},\frac{\vec{u}^{(3)}}{c}\right\rangle\right)\sigma_1^{(3)}\right]$$

$$+\frac{e_3^2}{m_3c^2}\left(-\frac{\left\langle \vec{u}^{(3)},\vec{\sigma}^{(3)}\right\rangle}{c^3}\ddot{u}_1^{(3)}+\frac{\left\langle \vec{\sigma}^{(3)},\ddot{\vec{u}}^{(3)}\right\rangle}{c^3}u_1^{(3)}+\frac{\left\langle \vec{u}^{(3)},\ddot{\vec{u}}^{(3)}\right\rangle}{c^4}\sigma_1^{(3)}\right)$$

$$\leq\approx \frac{|e_3e_1|}{m_3c^2}\left[\frac{4}{c^2}\frac{1}{{\tau_{31}}^2}+\frac{4e^{\mu T}\omega\beta}{\tau_{31}}\right]\sqrt{\sum_{\gamma=1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}$$

$$+\frac{|e_3e_2|}{m_3c^2}\left[\frac{4}{c^2}\frac{1}{{\tau_{32}}^2}+\frac{4e^{\mu T}\omega\beta}{\tau_{32}}\right]\sqrt{\sum_{\gamma=1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}+\frac{{e_3}^2\bar{c}e^{\mu T}\omega^2\bar{c}}{m_3c^2}\frac{2}{c^3}\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}$$

$$\leq\frac{|e_3e_1|}{m_3c^2}\left[\frac{4}{c^2{\tau_{31}}^2}+\frac{4e^{\mu T}\omega\beta}{\tau_{31}}+\frac{4}{c^2{\tau_{32}}^2}+\frac{4e^{\mu T}\omega\beta}{\tau_{32}}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right]\sqrt{\sum_{\gamma=1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}$$

$$\leq\frac{|e_3e_1|}{m_3c^2}\left[\frac{4}{{\Delta_{31}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{31}}+\frac{4}{{\Delta_{32}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{32}}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right]\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}$$

$$\left|H_2^{(3)}\right|\leq\frac{|e_3e_1|}{m_3c^2}\left[\frac{4}{{\Delta_{31}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{31}}+\frac{4}{{\Delta_{32}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{32}}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right]\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(3)}\right)^2};$$

$$\left|H_3^{(3)}\right|\leq\frac{|e_3e_1|}{m_3c^2}\left[\frac{4}{{\Delta_{31}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{31}}+\frac{4}{{\Delta_{32}}^2}+\frac{4\bar{c}e^{\mu T}\omega}{\Delta_{32}}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right]\sqrt{\sum_{\gamma-1}^{3}\left(\sigma_\gamma^{(3)}\right)^2}.$$

5. Existence Theorem for Spin Equations

Introduce the set of spin continuous functions

$$SP=\{\sigma(.)\in C[0,T]:|\sigma(t)|\leq S_0\wedge\sigma(0)=\sigma(T):t\in[0,T]\}$$

with a metric $\rho(\sigma,\bar{\sigma})=\sup\{|\sigma(t)-\bar{\sigma}(t)|:t\in[0,T]\}$.

We consider SP with a metric and define the Cartesian product:

$$(SP)^9=SP\times SP\times SP\times SP\times SP\times SP\times SP\times SP\times SP.$$

The problem of an existence of T-periodic solution of (11) is equivalent to the existence T-periodic solution of the integral equations (cf. [9]):

$$\sigma_\alpha^{(k)}(t)=\sigma_\alpha^{(k)}(T)+\int_0^t H_\alpha^{(k)}\left(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)ds, t\in[0,T],$$

($k=1,2,3;\alpha=1,2,3$), where $H_\alpha^{(k)}(t,\sigma_1,\sigma_2,\sigma_3)$ is T-periodic with respect to the first variable.

Now we define an operator with 9 components

$$B=\left(B_1^{(1)},B_2^{(1)},B_3^{(1)},B_1^{(2)},B_2^{(2)},B_3^{(2)},B_1^{(3)},B_2^{(3)},B_3^{(3)}\right):(SP)^9\to(SP)^9$$

by the right-hand sides of (11)

$$B_\alpha^{(k)}\left(\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)(t):=\sigma_\alpha^{(k)}(T)+\int_0^t H_\alpha^{(k)}\left(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)ds, t\in[0,T],(k=1,2,3;\alpha=1,2,3).$$

In what follows we prove an existence theorem for T-periodic solution of the spin equations.

Theorem 5.1. *Let the following conditions be fulfilled:*

$$|\rho_{k0}-\rho_{n0}|-(R_k+R_n)\frac{e^{\mu T}-1}{\mu}=\Delta_{kn}>0;$$
$$\left|\sigma_\alpha^{(1)}(T)\right|+\frac{4|e_1e_2|}{m_1c^2}\left(\frac{2}{\Delta^2}+\frac{2\bar{c}e^{\mu T}\omega}{\Delta}\right)\frac{e^{\mu T}-1}{\mu}\sqrt{3}\le 1;$$
$$\left|\sigma_\alpha^{(2)}(T)\right|+\frac{|e_2e_1|}{m_2c^2}\left(\frac{8}{\Delta^2}+\frac{8\bar{c}e^{\mu T}\omega}{\Delta}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right)\frac{e^{\mu T}-1}{\mu}\sqrt{3}\le 1;$$
$$\left|\sigma_\alpha^{(3)}(T)\right|+\frac{|e_3e_1|}{m_3c^2}\left[\frac{8}{\Delta^2}+\frac{8e^{\mu T}\omega\bar{c}}{\Delta}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right]\frac{e^{\mu T}-1}{\mu}\sqrt{3}\le 1,\ \textit{where}\ \Delta=\min\Delta_{kn}>0.$$

Then the system (11) has a continuous T-periodic solution.

Proof. We use the Arzela-Ascoli theorem and the Schauder fixed point theorem. It is easy to see that if the system has a solution belonging to $(SP)^9$, then integrating (11) we obtain

$$\sigma_\alpha^{(k)}(t)=\sigma_\alpha^{(k)}(0)+\int_0^t H_\alpha^{(k)}\left(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)ds, t\in[0,T]\Leftrightarrow \sigma_\alpha^{(k)}(t)$$
$$=\sigma_\alpha^{(k)}(T)+\int_0^t H_\alpha^{(k)}\left(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)(s)ds$$

and vice versa if the operator B has a fixed point belonging to $(SP)^9$ then after differentiating one obtains that the system $\frac{d\sigma_\alpha^{(k)}(t)}{dt}=H_\alpha^{(k)}\left(t,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right)$ has a solution belonging $(SP)^9$.

It is not difficulty to check that B maps $(SP)^9$ into itself:

$$\left|B_\alpha^{(1)}(t)\right|\le\left|\sigma_\alpha^{(1)}(T)\right|+\left|\int_0^t H_\alpha^{(1)}\left(s,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)ds\right|$$
$$\le\left|\sigma_\alpha^{(1)}(T)\right|+\frac{4|e_1e_2|}{m_1c^2}\left(\frac{2}{\Delta^2}+\frac{2\bar{c}e^{\mu T}\omega}{\Delta}\right)\frac{e^{\mu T}-1}{\mu}\sqrt{3}S_0\le S_0;$$

$$\left|B_\alpha^{(2)}(t)\right|\le\left|\sigma_\alpha^{(2)}(T)\right|+\left|\int_0^t H_\alpha^{(2)}\left(s,\sigma_1^{(2)},\sigma_2^{(2)},\sigma_3^{(2)}\right)ds\right|$$
$$\le\left|\sigma_\alpha^{(2)}(T)\right|+\frac{|e_2e_1|}{m_2c^2}\left(\frac{8}{\Delta^2}+\frac{8\bar{c}e^{\mu T}\omega}{\Delta}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right)\frac{e^{\mu T}-1}{\mu}\sqrt{3}S_0\le S_0;$$

$$\left|B_\alpha^{(3)}(t)\right|\le\left|\sigma_\alpha^{(3)}(T)\right|+\left|\int_0^t H_\alpha^{(3)}\left(s,\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}\right)ds\right|$$
$$\le\left|\sigma_\alpha^{(3)}(T)\right|+\frac{|e_3e_1|}{m_3c^2}\left[\frac{8}{\Delta^2}+\frac{8e^{\mu T}\omega\bar{c}}{\Delta}+\frac{2e^{\mu T}\omega^2\beta^2}{c}\right]\frac{e^{\mu T}-1}{\mu}\sqrt{3}S_0\le S_0.$$

Further on we have:

$$\left|B_\alpha^{(1)}\left(\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)(t)-B_\alpha^{(1)}\left(\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)(\bar{t})\right|$$
$$\le\left|\int_0^t H_\alpha^{(1)}\left(s,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)ds-\int_0^{\bar{t}} H_\alpha^{(1)}\left(s,\bar{\sigma}_1^{(1)},\bar{\sigma}_2^{(1)},\bar{\sigma}_3^{(1)}\right)ds\right|$$
$$\le\int_{\bar{t}}^t\left|H_\alpha^{(1)}\left(s,\sigma_1^{(1)},\sigma_2^{(1)},\sigma_3^{(1)}\right)\right|ds\le\frac{8|e_1e_2|}{m_1c^2}\left(\frac{1}{\Delta^2}+\frac{\bar{e}e^{\mu T}\omega}{\Delta}\right)\sqrt{3}S_0\frac{e^{\mu T}-1}{\mu}|t-\bar{t}|$$

$$\left|B_\alpha^{(2)}\left(\sigma_1^{(2)},\sigma_2^{(2)},\sigma_3^{(2)}\right)(t)-B_\alpha^{(2)}\left(\sigma_1^{(2)},\sigma_2^{(2)},\sigma_3^{(2)}\right)(\bar{t})\right|\le\int_{\bar{t}}^t\left|H_\alpha^{(2)}\left(s,\sigma_1^{(2)},\sigma_2^{(2)},\sigma_3^{(2)}\right)\right|ds$$
$$\le\frac{|e_2e_1|}{m_2c^2}\left(\frac{8}{\Delta^2}+\frac{8\bar{c}e^{\mu T}\omega}{\Delta}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right)\sqrt{3}S_0\frac{e^{\mu T}-1}{\mu}|t-\bar{t}|;$$

$$\left|B_{\alpha}^{(3)}\left(\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}\right)(t)-B_{\alpha}^{(2)}\left(\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}(\bar{t})\right)\right|\leq\int_{\bar{T}}^{t}\left|H_{\alpha}^{(3)}\left(s,\sigma_1^{(3)},\sigma_2^{(3)},\sigma_3^{(3)}\right)\right|ds$$
$$\leq\frac{|e_3e_1|}{m_2c^2}\left(\frac{8}{\Delta^2}+\frac{8\bar{c}e^{\mu T}\omega}{\Delta}+\frac{2e^{\mu T}\omega^2\bar{c}^2}{c^3}\right)\sqrt{3}S_0\frac{e^{\mu T}-1}{\mu}|t-\bar{t}|.$$

Therefore, applying the Arzela-Ascoli theorem and the Schauder fixed point theorem we conclude that operator B has a fixed point which is a periodic solution of (11).

The main theorem is thus proved. □

6. Conclusion

As in the previous papers we stick to the Corben's formalism (cf. [10, 11]) and note some previous papers devoted to the classical spin theory [12, 13, 14, 15, 16]. For the concrete case we recall the inequalities

$$\Delta_{kn}=|\rho_{k0}-\rho_{n0}|-(R_k+R_n)\frac{e^{\mu\pi}-1}{\mu}>0$$

and

$$\rho_{n0}-R_n\frac{e^{\mu\pi}-1}{\mu}>0;(n=2,3)$$

It is easy to verify that if $\left(\sigma_1{}^{(k)},\sigma_2{}^{(k)},\sigma_3{}^{(k)}\right)$ is a solution of the above system then $\left(-\sigma_1{}^{(k)},-\sigma_2{}^{(k)},-\sigma_3{}^{(k)}\right)$ is a solution, too. Consequently, one can conclude that two particles with opposite spin are possible configuration but not compulsory ones.

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