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A short note on the article "Some Applications of the Weak Contraction Principle"

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Abstract

The purpose of this brief note is to correct and clarify a wrong assertion given in the recent article "Some Applications of the Weak Contraction Principle" [Lett. Nonlinear Anal. Appl. 3 (2025), 1-4], that affects both a valid example used in the article "Remarks on the fixed point theory for quasi-metric spaces" [Results Nonlinear Anal. 7 (2024), 70–74], and two incorrect results presented in the article "All metric fixed point theorems hold for quasi-metric spaces" [Results Nonlinear Anal. 6 (2023), 116–127].

Keywords: Quasi-metric space; fixed point.
2010 MSC: 54H25, 54E50, 47H10

In [1], Park obtained various interesting fixed point results in the framework of quasi-metric spaces. In [3], it was given an example that contradicts Theorem 6.3 and Corollary 6.5 of [1]. Recently, Park [2, Section 3] claimed that our example does not invalidate the aforementioned results. The purpose of this brief note is to correct Park's claim.

The example (see [3, p. 72] or [2, p. 3]). Let $X = \mathbb{N} \cup \{\infty\}$ and let d be the quasi-metric on X given by $d(x, x) = 0$ for all $x \in X$, $d(n, \infty) = 0$ for all $n \in \mathbb{N}$, $d(\infty, n) = 1/n$ for all $n \in \mathbb{N}$, and $d(n, m) = 1/m$ for all $n, m \in \mathbb{N}$ with $n \neq m$.

Now let T be the self-mapping of X defined as $T\infty = 1$, and $Tn = 2n$ for all $n \in \mathbb{N}$.

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Then, we have

$$d(T\infty, T^2\infty) = d(1,2) = \frac{1}{2} = \frac{1}{2}d(\infty,1) = \frac{1}{2}d(\infty, T\infty),$$

and, for each $n \in \mathbb{N}$,

$$d(Tn, T^2n) = d(2n, 4n) = \frac{1}{4n} = \frac{1}{2}d(n, 2n) = \frac{1}{2}d(n, Tn).$$

Therefore, all the conditions of Theorem 6.3 in [1] are satisfied for $\alpha = 1/2$ (actually, for any constant $\alpha \in [1/2, 1)$). However, T has no fixed points.

Consequently, Theorem 6.3 and Corollary 6.5 in [1] (Corollaries P.1 and P.2 in [2]) are not true, as already stated in [3].

Finally, note that the error in Park's argument occurs on line -3 of page 3. The fact that $\alpha \geq 1/2$ does not imply that "In any case, it violates $0 < \alpha < 1$ in Corollary P.1".

References

[1] S. Park, *All metric fixed point theorems hold for quasi-metric spaces*, Results Nonlinear Anal. **6** (2023), 116–127.
[2] S. Park, *Some applications of the Weak Contraction Principle*, Lett. Nonlinear Anal. Appl. **3** (2025), 1-4.
[3] S. Romaguera, *Remarks on the fixed point theory for quasi-metric spaces*, Results Nonlinear Anal. Appl. **7** (2024), 70–74.